Robust Control of F-16 Lateral Dynamics

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***Abstract*— We consider the application of a conditional integrator based continuous sliding mode control design for robust regulation of MIMO minimum-phase nonlinear systems to the control of the lateral flight dynamics of an F-16 aircraft. The system is non-affine in the input but can be rewritten as the perturbation of a control affine system with matched (input-dependent) disturbances. A parameter dependent transformation brings the system to normal form, for which an output-feedback control can be designed to achieve robust regulation. We provide analytical results for stability, and also show through extensive simulations that the inherent robustness of the SMC design provides a convenient way to design controllers without adaptation for the unknown parameters, with a transient performance that is comparable to discontinuous SMC, but without suffering from the drawback of control chattering.**

I. INTRODUCTION

The dynamic response characteristics of aircraft are highly nonlinear. Traditionally, flight control systems have been designed using mathematical models of the aircraft linearized at various flight conditions, with the controller parameters or gains "scheduled" or varied with the flight operating conditions. Various robust multivariable techniques including linear quadratic optimal control (LQR/LQG), H_∞ control, and structured singular value μ-synthesis have been employed in controller design, an excellent and exhaustive compendium of which is available in [9]. In order to guarantee stability and performance of the resulting gain-scheduled controllers, analytical frameworks of gain scheduling have been developed, including the powerful technique of linear-parameter-varying (LPV) control [3], [8], [18], [21]. Nonlinear design techniques such as dynamic inversion have been used in [1], [11], [20], while a technique that combines model inversion control with an online adaptive neural network to "robustify" the design is described in [12], and a nonlinear adaptive design based on backstepping and neural networks in [7]. A radial basis functions neural network (RBFNN) based adaptive design with time-scale separation between the system and controller dynamics, with applications to control of both longitudinal (angle-of-attack command systems) as well as lateral (regulation of the sideslip and roll angles) dynamics is described in [22]. A succinct "industry perspective" on flight control design, including the techniques of robust control (H_∞, μ-synthesis), LPV control, dynamic inversion, adaptive control, neural networks, and more, can be found in [2].

Our interest is in the design of robust sliding mode control (SMC) for the lateral flight dynamics of a F-16 aircraft. More specifically, we wish to design the aileron and rudder controls to asymptotically track desired references for the side-slip and roll angles. The application of SMC to flight control has been pursued by several others authors, see, for example, [4], [5], [19]. Our work differs from earlier ones in that it is based on a recent technique in [14], [16] for introducing integral action in SMC, that we refer to as "conditional integrators". The controller that we present has a very simple structure; aside from an "input-decoupling" term, it is simply two saturated high-gain PID controllers, with the (anti-windup) conditional integrator, and the derivative estimated by a high-gain observer. This controller structure is a special case of a general design for robust output regulation for multiple-input multiple-output (MIMO) nonlinear systems transformable to the normal form, with analytical results for stability and performance described in [14], [16]. The inclusion of integral control in SMC design on the one hand allows us to use smaller gains, while the conditional design means that integral action takes place only inside the boundary layer, allowing us to recover the transient performance of ideal (discontinuous) SMC, but without suffering from the trade-off between tracking accuracy and robustness to unmodeled high-frequency dynamics.

The rest of this paper is organized as follows. In Section 2, we describe the nonlinear mathematical aircraft model, while in Section 3, we present our controller design, based on the results in [14], [16]. Simulation results showing the efficacy of the design, including demonstrating the robustness to parameter uncertainties, disturbances and time-delays, are presented in Section 4. Finally, a summary of our work and some suggestions for possible extensions are provided in Section 5.

II. AIRCRAFT LATERAL MODEL

The model of the lateral dynamics that we use for control design is described below, and is extracted practically verbatim from [22].

$$\left.\begin{array}{rcl} \dot{\phi} &=& \frac{\cos\gamma_0}{\cos\theta_0}p_s + \frac{\cos\gamma_0}{\cos\theta_0}r_s \\ \dot{\beta} &=& \frac{Y_\beta}{V}\beta + \frac{Y_r}{V}r_s + \frac{g\cos\theta_0}{V}\phi - r_s \\ \dot{p_s} &=& L_\beta\beta + L_p p_s + L_r r_s + \delta_l(p_s, r_s) + \\ && L_{\delta_a}(\beta,\delta_a) + L_{\delta_r}(\beta,\delta_r) \\ \dot{r_s} &=& N_\beta\beta + N_p p_s + N_r r_s + \delta_n(p_s, r_s) + \\ && N_{\delta_a}(\beta,\delta_a) + N_{\delta_r}(\beta,\delta_r) \end{array}\right\} \quad (1)$$

The system has 4 states ϕ, β, p_s and r_s, which are respectively the roll angle, the sideslip, and the stability axis roll and yaw rates, and two inputs δ_a and δ_r, which

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are respectively the aileron and rudder control. The other variables in (1) are the gravitational constant g, the trimmed pitch angle θ_0, the trimmed flight path angle γ_0, the true airspeed V, the aerodynamic stability and control derivatives Y_β and Y_r (which can be taken as being approximately constant), the incremental rolling and yawing moments δ_l and δ_n that are unknown functions of the roll and yaw rates, and the rolling and yawing moments due to aileron and rudder deflections $L_{\delta_a}, L_{\delta_r}, N_{\delta_a}, N_{\delta_r}$. Following [22], we assume that the nonlinear terms $\delta_l(p_s, r_s)$ + $L_{\delta_a}(\beta, \delta_a)$ + $L_{\delta_r}(\beta, \delta_r)$ and $\delta_n(p_s, r_s)$ + $N_{\delta_a}(\beta, \delta_a)$ + $N_{\delta_r}(\beta, \delta_r)$ in (1) can be expressed as shown below

$$\begin{aligned} \delta_l + L_{\delta_a} + L_{\delta_r} &= L_{\delta_{a_0}}(\delta_a + f_1(\cdot)) + L_{\delta_{r_0}}(\delta_r + f_2(\cdot)) \\ \delta_n + N_{\delta_a} + N_{\delta_r} &= N_{\delta_{a_0}}(\delta_a + f_1(\cdot)) + N_{\delta_{r_0}}(\delta_r + f_2(\cdot)) \end{aligned} \tag{2}$$

where the “linear terms" $L_{\delta_{a_0}}$, $L_{\delta_{r_0}}$, $N_{\delta_{a_0}}$, and $N_{\delta_{r_0}}$ are known (i.e., we use nominal values for these parameters) and all uncertainty is lumped into the nonlinear functions $f_1(\beta, p_s, r_s, \delta_a)$ and $f_2(\beta, p_s, r_s, \delta_r)$. Consequently, system (1) can be written compactly in standard state-space form as

$$\begin{aligned} \begin{bmatrix} \dot\beta \\ \dot\phi \\ \dot p_s \\ \dot r_s \end{bmatrix} &= A \begin{bmatrix} \beta \\ \phi \\ p_s \\ r_s \end{bmatrix} + B \begin{bmatrix} \delta_a + f_1(\beta, p_s, r_s, \delta_a) \\ \delta_r + f_2(\beta, p_s, r_s, \delta_r) \end{bmatrix} \\ y &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ \phi \\ p_s \\ r_s \end{bmatrix} \stackrel{\text{def}}{=} Cx = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x \end{aligned} \tag{3}$$

where

$$A = \begin{bmatrix} \frac{Y_\beta}{V} & \frac{g\cos\theta_0}{V} & 0 & -1 \\ 0 & 0 & \frac{\cos\gamma_0}{\cos\theta_0} & \frac{\sin\gamma_0}{\cos\theta_0} \\ L_\beta & 0 & L_p & L_r \\ N_\beta & 0 & N_p & N_r \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{\delta_{a_0}} & L_{\delta_{r_0}} \\ N_{\delta_{a_0}} & N_{\delta_{r_0}} \end{bmatrix} \tag{4}$$

In [22], it is assumed that based on available wind-tunnel data, the unknown nonlinear functions $f_1(\cdot)$ and $f_2(\cdot)$ have the following form:

$$\begin{aligned} f_1(\cdot) &= ((1 - E_1)e^{-\frac{\beta^2}{2\delta_1{}^2}} + E_1)(\tanh(\delta_a + h_1) \\ &+ \tanh(\delta_a - h_1) + 0.001\delta_a) \\ &+ D_1\cos(A_1 p_s - w_1)\sin(A_2 r_s - w_2) + D_2 \end{aligned} \tag{5}$$

$$\begin{aligned} f_2(\cdot) &= ((1 - E_2)e^{-\frac{\beta^2}{2\delta_2{}^2}} + E_2)(\tanh(\delta_r + h_2) \\ &+ \tanh(\delta_r - h_2) + 0.001\delta_r) \\ &+ D_3\cos(A_3 p_s - w_3)\sin(A_4 r_s - w_4) + D_4 \end{aligned} \tag{6}$$

A physical interpretation of the constants $0 < E_{1,2} < 1$ as the percentage of the control effectiveness available at high angles of sideslip is discussed in [22]. As opposed to [22] where the design is based on adaptive approximation using RBFNNs, we do not require to know the specific form of the functions in our design, except to (numerically) verify an assumption that we will make later on.

III. Control Design

Our control objective is to design a control signal $\delta = \begin{bmatrix} \delta_a(t) \\ \delta_r(t) \end{bmatrix}$ such that the output $y(t) = \begin{bmatrix} \beta(t) \\ \phi(t) \end{bmatrix}$ tracks a smooth commanded reference input $y_{ref}(t) = \begin{bmatrix} \beta_{ref}(t) \\ \phi_{ref}(t) \end{bmatrix}$, robustly in the presence of roll/yaw parametric uncertainties and aileron/rudder unknown control effects.

Our approach to control design is based (see [14], [16]) on minimum-phase systems transformable to the normal form. To simplify the presentation, we start with the SISO control-affine case of a nonlinear system with full relative degree $\rho = n$ (i.e., with no zero-dynamics), that can be transformed to the normal form

$$\dot\xi = A_c\xi + B_c\ [a(\xi) + b(\xi)u],\ \ y = C_c\xi$$

where $\xi \in R^\rho$ the output and its derivatives up to order $\rho - 1$, and the triple (A_c, B_c, C_c) a canonical form representation of a chain of ρ integrators, and the functions $a(\cdot)$ and $b(\cdot)$ are unknown, but $b(\cdot)$ is globally bounded away from zero, and a lower bound on its magnitude is known. Under the assumption that all exogenous signals (that include the reference $y_{ref}(t)$) are asymptotically constant, we proceed with an integral control based design. In particular, for the class of systems considered here, it is shown in [14], [16] that a continuous sliding mode controller of the form

$$u = -k\text{sign}(b(\cdot))\ \text{sat}\left(\frac{k_0\sigma + k_1 e_1 + k_2 e_2 + \cdots + e_\rho}{\mu}\right) \tag{7}$$

can be designed to achieve robust regulation, where $e_1 = y - y_{ref}$ is the tracking error, and e_2, ..., e_ρ are its derivatives up to order ρ, the positive constants $k_i, i = 1, \cdots, \rho - 1$ in the sliding surface function

$$s = k_0\sigma + \sum_{i=1}^{\rho} k_i e_i + e_\rho \tag{8}$$

are chosen such that the polynomial

$$\lambda^{\rho-1} + k_{\rho-1}\lambda^{\rho-2} + \cdots + k_1$$

is Hurwitz, and σ_i is the output of the “conditional integrator"

$$\dot\sigma = -k_0\sigma + \mu\ \text{sat}\left(\frac{s}{\mu}\right),\ \ \sigma(0) \in [-\mu/k_0, \mu/k_0] \tag{9}$$

where $k_0 > 0$, and $\mu > 0$ is the “width" of the boundary layer. From (8) and (9), it is clear that inside the boundary layer $|s| \le \mu$,

$$\dot\sigma = k_1 e_1 + k_2 e_2 + \cdots + e_\rho$$

which implies that $e_i = 0$ at equilibrium, i.e., (9) is the equation of an integrator that provides integral action “conditionally", inside the boundary layer. As shown in [14], [16], such a design provides asymptotic error regulation, while not degrading the transient performance, as is common in a conventional design that uses the integrator $\dot\sigma = e_1$. In the case of relative degree $\rho = 1$ and $\rho = 2$, the controller (7) is simply a specially tuned saturated PI/PID controller with anti-windup (see [16, Section 6]).

The control (7) can be extended to the output-feedback case [1] e_i by its estimate $\hat{e}_i$, obtained using the high-gain observer (HGO)

$$\left.\begin{array}{rcl}\dot{\hat{e}}_i &=& \hat{e}_{i+1} + \alpha_i(e_1 - \hat{e}_1)/\epsilon^i,\ 1 \le i \le \rho - 1 \\ \dot{\hat{e}}_\rho &=& \alpha_\rho(e_1 - \hat{e}_1)/\epsilon^\rho\end{array}\right\} \quad (10)$$

where $\epsilon > 0$, and the positive constants α_i are chosen such that the roots of

$$\lambda^\rho + \alpha_1\lambda^{\rho-1} + \cdots + \alpha_{\rho-1}\lambda + \alpha_\rho = 0$$

have negative real parts. To complete the design, we need to specify how k, μ and ϵ (in the output-feedback case) are chosen. The parameter k is chosen "sufficiently large" (to overbound uncanceled terms in $\dot{s}$) while μ and ϵ are chosen "sufficiently small", the former to recover the performance of ideal (discontinuous) SMC (without an integrator) and the latter to recover the performance under state-feedback with the continuous SMC. Analytical results for stability and performance are given in [14], [16].

While we abstracted the design from [14], [16] above for SISO systems for the sake of clarity of presentation, the design in those papers was done for MIMO systems. Consequently, our next step is to extend the design presented above for the MIMO system (3), keeping in mind the basic features presented above. To that end, we first rewrite

$$\begin{array}{rcl}f_1(\cdot,\delta_a) &=& f_1(\cdot,0) + \delta_{f_1}(\cdot,\delta_a) \\ f_2(\cdot,\delta_r) &=& f_2(\cdot,0) + \delta_{f_2}(\cdot,\delta_r)\end{array}$$

and define $f(\cdot,0) \stackrel{\text{def}}{=} \begin{bmatrix} f_1(\cdot,0) \\ f_2(\cdot,0)\end{bmatrix}$. Then, it is easy to check that with $\delta_{f_i}(\cdot) \equiv 0$, the control-affine system $\dot{x} = Ax + B[u + f(\cdot,0)]$ has strong vector relative degree $\rho = \{2,2\}$, since $C_1B = C_2B = 0$ and $T = CAB$ is nonsingular, and that with $\delta_{f_i}(\cdot) \neq 0$, the change of variables

$$e_{1\beta} = \beta - \beta_{ref},\ e_{2\beta} = \dot{e}_{1\beta},\ e_{1\phi} = \phi - \phi_{ref},\ e_{2\phi} = \dot{e}_{1\phi}$$

transforms the system to a normal form very similar to the one in the SISO case, that is identical to the general case considered in [14]. In particular, it can be verified that the equations for $\dot{e}_{2\beta}$ and $\dot{e}_{2\phi}$ take the form

$$\dot{e}_{2z} = b_z(x) - \ddot{z}_{ref} + t_{i1}(\delta_a + \delta_{f_1}(\cdot)) + t_{i2}(\delta_r + \delta_{f_2}(\cdot)),\ z = \beta,\phi$$

where $T = CAB = \{t_{ij}\}$. As done in the SISO case, for $z = \beta,\phi$, we define

$$\left.\begin{array}{rcl} s_z &=& k_{z0}\sigma_z + k_{z1}e_{1z} + e_{2z} \\ \dot{\sigma}_z &=& -k_{z0}\sigma_z + \mu_z \text{ sat}\left(\frac{s_z}{\mu_z}\right)\end{array}\right\} \quad (11)$$

where the second set of equations represent conditional integrators that provide integral action only inside the boundary layers $|s_z| \le \mu_z$. When $\delta_{f_i}(\cdot) = 0$, a standard sliding mode argument (see, for example, [16]) shows that the control

$$\delta = T^{-1}v \stackrel{\text{def}}{=} T^{-1}\begin{bmatrix} v_\beta \\ v_\phi\end{bmatrix},\ v_z = -\gamma_z \text{ sat}\left(\frac{s_z}{\mu_z}\right) \quad (12)$$

achieves semi-global regulation, provided the gains γ_β and γ_ϕ can be chosen "sufficiently large", and the boundary layer widths μ_β and μ_ϕ are chosen "sufficiently small". More precise analytical results for stability and performance of the above control design in the control-affine case (i.e., $\delta_{f_i}(\cdot) = 0$) can be found as [16, Theorems 1 & 2].

With $\delta_{f_i}(\cdot) \neq 0$, which really is the case of interest here, an additional assumption is required, which we state below. To see the motivation for this assumption, we differentitate (11) and with the control as defined in (12), we rewrite the resulting equation in the form

$$\dot{s}_z = \Delta_z(x,\sigma,e,\ddot{z}_{ref},v) - \gamma_z \text{ sat}\left(\frac{s_z}{\mu_z}\right) \quad (13)$$

where $\sigma = \begin{bmatrix}\sigma_\beta \\ \sigma_\phi\end{bmatrix}$, and $e = \begin{bmatrix} e_\beta \\ e_\phi\end{bmatrix}$, with $e_z = \begin{bmatrix} e_{1z} \\ e_{2z}\end{bmatrix}$, $z = \beta,\phi$.

From (13), it is clear that for the control term $v_z = -\gamma_z\text{sat}\left(\frac{s_z}{\mu_z}\right)$ to "overbound" the term $\Delta_z(x,\sigma,e,\ddot{z}_{ref},v)$, which in turn depends on both v_β and v_ϕ, some restriction needs to be imposed on the way $\Delta_z(\cdot,v)$ depends on v. The next assumption states this restriction precisely.

Assumption 1:

$$max\left|\frac{\Delta_z(\cdot)}{\gamma_z}\right| \le \varrho_z + \kappa_{z1}|v_\beta| + \kappa_{z2}|v_\phi|,\ z = \beta,\phi$$

where the maximization is taken over some compact set $\mathcal{S}$ of interest, the constants [2] ϱ_z and κ_{zi} are known and $I - K$, where $K = \begin{bmatrix}\kappa_{\beta1} & \kappa_{\beta2} \\ \kappa_{\phi1} & \kappa_{\phi2}\end{bmatrix}$, is an M-matrix.

The constants γ_z in the control (12) are chosen as follows. The fact that $I - K$ is an M-matrix implies that (i) $(I-K)$ is nonsingular, (ii) the elements of $(I-K)^{-1}$ are non-negative, and (iii) there exists a vector l with $l_z > 0$ such that the elements of $b = (I-K)l$ are all positive. For $z = \beta,\phi$, let $\bar{\varrho} = \begin{bmatrix}\bar{\varrho}_\beta \\ \bar{\varrho}_\phi\end{bmatrix}$, with $\bar{\varrho}_z \ge \varrho_z$, and take $\gamma_z = \psi_z + l_z$, where $\psi = \begin{bmatrix}\psi_\beta \\ \psi_\phi\end{bmatrix} = (I-K)^{-1}\bar{\varrho}$. With this choice of γ_z, the analysis in [14] shows that the (state-feedback) control (11), (12) achieves robust regional regulation, with $\mathcal{S}$ a subset of the region of attraction. The design is then extended to the output-feedback case by replacing $e_{2z} = \dot{e}_{1z}$, $z = \beta,\phi$, in (11) (and hence also in the control (12)) with their estimates using HGOs. While we have not verified Assumption 1 rigorously, our simulation results, which we present next, validate (the assumption and) our control design.

IV. Simulation Results

For our simulations, we use the same values of the aircraft parameters that are used in [22]. In particular, for the trim

[1]This might be required even when the original state x is available, since ξ and hence e_i, which is required in the control, depend on the state through possibly unknown parameters.

[2]The function ϱ_z can be chosen to be error dependent, and is assumed constant only for convenience. It can always be chosen to be constant over compact sets, but at the cost of conservatism.

values corresponding to an airspeed of $V = 502 ft/s$ and angle of attack $\alpha = 2.11°$, we have

$$A = \begin{bmatrix} -0.3220 & 0.0640 & 0.0364 & -0.9917 \\ 0 & 0 & 1 & 0.0393 \\ -30.6490 & 0 & -3.6784 & 0.6646 \\ 8.5395 & 0 & -0.0254 & -0.4764 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -0.7331 & 0.1315 \\ -0.0319 & -0.0620 \end{bmatrix}$$

The values of the constants that appear in the expressions for $f_1(\cdot)$ and $f_2(\cdot)$ are computed using averaged (and curve-fitted) wind-tunnel data for $\alpha = 0°$ and $\alpha = 5°$, with the curve fit approximation done over $\beta \in [-30°, 30°]$, $p_s \in [-180°, 180°]$, $r_s \in [-90°, 90°]$, $\delta_a \in [-21.5°, 21.5°]$, and $\delta_r \in [-30°, 30°]$. From [22], the corresponding values for the constants then are $A_1 = A_2 = A_3 = A_4 = 0.1$, $D_1 = 0.075, D_2 = 0.0016, D_3 = 0.45, D_4 = 0$, $\omega_1 = \omega_3 = 1.5$, $\omega_2 = \omega_4 = 0$, $E_1 = E_2 = 0.3$, $h_1 = 7, h_2 = 4$, $\sigma_1 = 0.015, \sigma_2 = 0.15$. The commanded reference input is taken as

$$\begin{bmatrix} \beta_{ref}(t) \\ \phi_{ref}(t) \end{bmatrix} = \begin{bmatrix} 0.2\left(-\frac{0.5}{1+e^{t-8}} + \frac{1}{1+e^{t-30}} - 0.5\right) \\ 0.2\left(-\frac{0.5}{1+e^{t-8}} + \frac{1}{1+e^{t-30}} - 0.2\right) \end{bmatrix}$$

All initial conditions are taken as zero. Throughout, we assume the (nominal) value of the decoupling matrix $T = CAB = \begin{bmatrix} 0.005 & 0.0663 \\ -0.7344 & -0.1291 \end{bmatrix}$, and for the controller parameters, $k_{\beta 0} = k_{\phi 0} = k_{\beta 1} = k_{\phi 1} = 5$, and $k_\beta = k_\phi = 10$ [3]. The numerical values of the HGO parameters (for both the HGOs) are chosen as $\epsilon = 0.1$, $\alpha_1 = 15$, and $\alpha_2 = 50$. For clarity of presentation, we consider the following three cases:

A. The linear case: $\dot{x} = Ax + Bu$, $f_i(\cdot) \equiv 0$

Our first simulation shows the performance of the control designed in Section 3, but without integral action and when there are no actuator dynamics. Since there is no need to use SMC when the parameters are exactly known, in order to make the simulation meaningful, we assume that the values of A and B are randomly perturbed from their nominal values by 20%. The results are shown in Figure 1, for 2 different values of both μ_β and μ_ϕ, namely $\mu = 1$ and $\mu = 0.1$. From the figure, it is clear that the controller achieves good performance, even without integral control. The figure also shows that the errors are non-zero without integral control, but decrease as μ decreases.

The previous simulation shows that in the absence of integral control, $|e| = O(\mu)$, and we must decrease μ in order to achieve smaller steady-steady state errors, and this is clear from the simulation results. However, smaller values of μ can induce chattering when there are switching imperfections such as delays or unmodeled actuator dynamics. On the other hand, the inclusion of integral action means that we don't need to make μ very small to achieve small errors, only small enough to stabilize the equilibrium point. In order to emphasize this, we repeat the previous simulation, but assume first order actuator lag dynamics with time constant $\tau = 20.2 sec$, i.e., $H(s) = \frac{20.2}{s+20.2}$. The simulation results are shown in Figure 2, and it is clear that there is chattering in the control as μ is made small in order to obtain smaller errors[4].

[3]In addition to the saturated values of k_β and k_ϕ of v_β and v_ϕ respectively, we also saturate the controls δ_a and δ_r at their physically allowable limits of 21.5° and 30°.

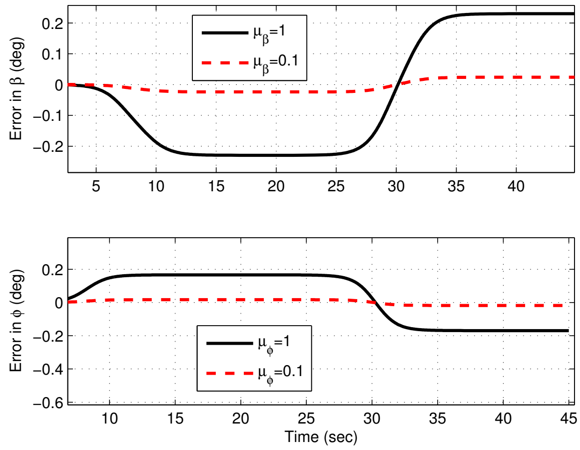

Fig. 1. Tracking errors without actuator dynamics and no integral control.

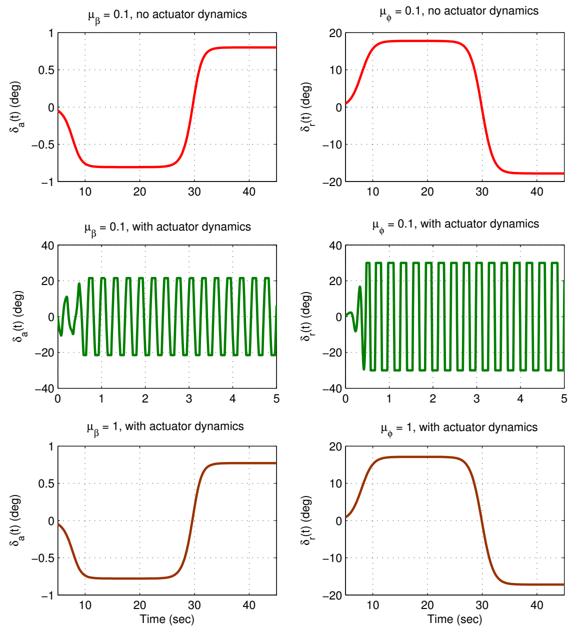

Fig. 2. Effect of decreasing μ in the presence of (unmodeled) actuator dynamics.

[4]Robustness of the conditional integrator based SMC design to time delays has also been demonstrated by simulations for the case of control of longitudinal dynamics (specifically the pitch-rate) of an F-16 in a related paper [10].

As mentioned before, with integral control, we don't need to decrease μ in order to obtain smaller errors. To that end, we repeat previous simulation with $\mu = 1$, but with the conditional integrator, and the results shown in Figure 3. It is clear that the inclusion of integral control makes the error asymptotically converge to zero.

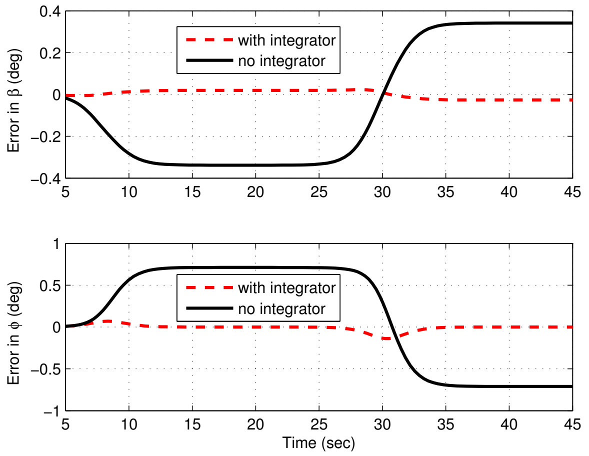

Fig. 3. Recovery of asymptotic regulation with conditional integrator.

It is important to note that the results presented above are only valid under output feedback. In particular, if one uses the original state x, then the disturbances are no longer matched when A is perturbed (any perturbation in B clearly constitutes a matched disturbance), so that if one uses $\dot{e}_{1\beta} = C_1Ax - \dot{\beta}_{ref}$ to compute $e_{2\beta}$ and use it in the control (and similarly for $\dot{e}_{1\phi}$), then the error will not be zero even with integral control when A is different from its nominal value (since our design explicitly uses the fact that the disturbances are matched). However, with the transformation to normal form, the disturbance is matched, and it is well-known (see, for example, [6, Chapter 14]) that the HGO can be used for this class of systems (where the states are the output and its derivatives) to achieve asymptotic error regulation. A discussion of the effect of measurement noise on the HGO is discussed in our related work in [17] on the control of F-16 longitudinal dynamics, where we show that the performance is not degraded significantly when the HGO is used with noisy measurements, and so we do not repeat similar simulations in this current work.

B. The control affine case: $\dot{x} = Ax + B(u + f(\cdot, 0))$

As shown in Section 3, the controller for this case has the exact same structure as that in the preceding case. Consequently, we simply repeat some of the simulations in the preceding subsection, with some minor differences. There is no need to perturb A and B in this case, since now the system already contains the unknown functions $f(\cdot, 0)$. Figure 4 shows the tracking performance in the presence of the unknown functions $f(\cdot, 0)$, and it is clear that the controller achieves good performance, and that the errors asymptotically converge to zero with the integral control.

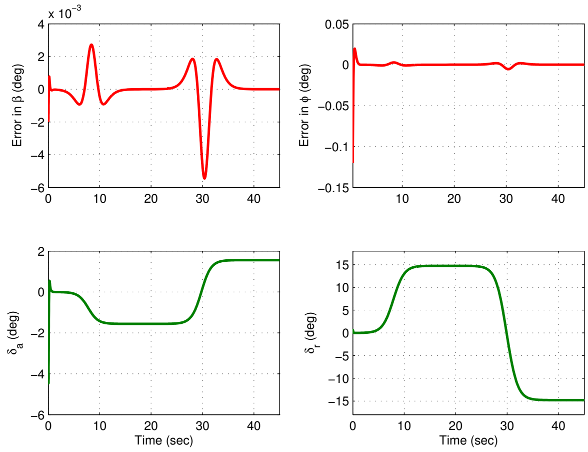

Fig. 4. Asymptotic regulation in control-affine case with unknown nonlinearities $f(\cdot, 0)$.

C. The control non-affine case: $\dot{x} = Ax + B(u + f(\cdot, u))$

When the function $f(\cdot)$ depends on u, we need to verify Assumption 1, and design the gains γ_z, $z = \beta, \phi$ accordingly. As mentioned in the preceding section, we do not rigorously verify (numerically) that this assumption holds in our work, but simply choose the gains to be their physically maximum allowable value, and demonstrate the efficacy of the design through simulations. To do so, we repeat the simulation of the previous subsection, but now with $f(\cdot, u)$, and not just $f(\cdot, 0)$. We also do so with several different initial conditions, with the simulation results plotted in Figure 5. The figure clearly shows that the initial error is larger as we move from the first row to the last (the initial conditions for β and ϕ were chosen that way) but that the controller achieves good performance in the presence of the control-dependent unknown nonlinearities $f(\cdot, u)$.

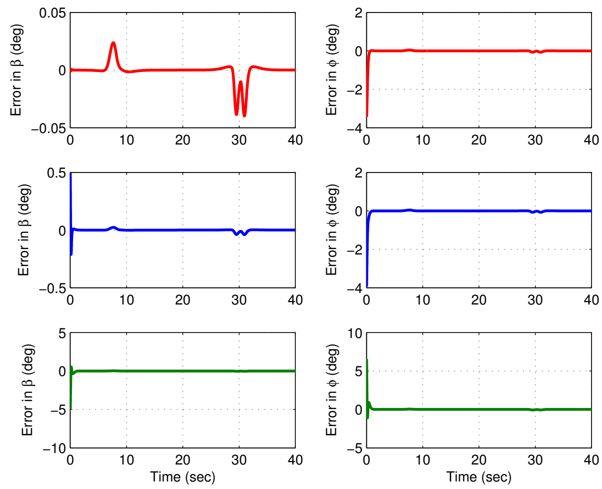

Fig. 5. Asymptotic regulation in control non-affine case with unknown nonlinearities $f(\cdot, u)$ for several different initial conditions.

Before we present our conclusions, we mention again that our controller has a very simple structure, that of a saturated PID controller with "anti-windup" like integrator, and an input-decoupling matrix. The only precise knowledge that the controller requires is the relative degree of the system and the signs of the high-frequency gains. It is robust to unmodeled actuator lag dynamics, and also to time-delays and measurement noise (as demonstrated in related work). For the non-affine in the control case, our simulation results are comparable to the ones presented in [22], where the controller is much more complex, and uses RBFNNs in conjunction with time-scale separation in an adaptive control design. We note that the design presented in this paper is flexible enough to allow for error-dependent or time-varying gains γ_z and also allow cancellation of any known/nominal terms in $\dot{s}$. This is elaborated upon in both [14], [16], but we did not mention it here for clarity of presentation. Finally, the design presented here is valid for a more general class of nonlinear systems than the F-16 lateral dynamics considered in this work, and has been successfully applied to the control of machines [15], process control [13], and F-16 longitudinal dynamics [10], [17].

V. Conclusions

We have presented a new SMC design for control of the lateral dynamics of an F-16 aircraft, based on the conditional integrator design of [14], [16]. The idea is based on rewriting the non-affine in the input system as a perturbation of a control-affine system, with guaranteed analytical results for stability and performance. The robustness of the method to modeling uncertainties and unmodeled actuator lag dynamics was demonstrated through simulation, with the transient and steady-state performance comparable to that of a more complex adaptive controller. In related work, we have shown the robustness of the design to time-delays and measurement noise (for the output-feedback case) [5]. Consequently, we believe that the results presented in this paper are a promising start to demonstrate the efficacy of the conditional integrator based SMC design to flight control.

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