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Robust output feedback regulation of minimum-phase nonlinear systems using conditional integrators☆

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Abstract

This paper is on the design of robust output feedback integral control for minimum-phase nonlinear systems with a well-defined relative degree. Previous work has shown how to design such controllers to achieve asymptotic regulation by a two-step process. First, robust control is designed to bring the trajectories to a small neighborhood of an equilibrium point. Within this neighborhood, the control then acts as a high-gain feedback that stabilizes the equilibrium point.

The asymptotic regulation achieved by integral action happens at the expense of degrading the transient performance. In this paper, we present an approach to improve the transient performance. The control design is a continuous sliding mode control with integral action. However, the integrator is introduced in such a way that it provides integral action only "conditionally", effectively eliminating the performance degradation. There are two main results in the paper: the first is asymptotic regulation and the second confirms the transient performance improvement by showing that the output feedback continuous sliding-mode control with integral action can be tuned to recover the performance of a state feedback ideal sliding mode control without integral action.

Keywords: Output regulation; Integral control; Minimum-phase systems; High-gain observers; Sliding-mode control

1. Introduction

We consider the problem of robust output regulation for multi-input multi-output (MIMO) minimum-phase nonlinear systems transformable into the normal form, uniformly in a set of constant disturbances and uncertain parameters. For this class of systems, robust continuous feedback control can be designed using techniques like high-gain feedback, min–max control, or sliding-mode control (SMC), to ensure convergence of the tracking error to a neighborhood of the origin, while rejecting bounded disturbances. However, making the error small requires the use of high-gain feedback near the origin. Typically, the steady-state error is of an order inversely proportional to the high-gain. To get smaller errors, it is therefore necessary to increase the gain, which is undesirable because it can excite unmodeled high-frequency dynamics. Moreover, in the case of min–max and sliding-mode control, where a continuous controller is obtained by approximating a discontinuous one, trying to make the approximation arbitrarily close results in chattering due to delays or unmodeled dynamics (Young, Utkin, & Ozguner, 1999). There is thus a trade-off between tracking accuracy and robustness to high-frequency unmodeled dynamics.

For constant or eventually constant exogenous signals, we can achieve zero steady-state error by introducing integral action in the controller (Byrnes, Priscoli, & Isidori, 1997; Huang & Rugh, 1992; Isidori, 1997; Isidori & Byrnes, 1990; Khalil, 1994, 2000; Mahmoud & Khalil, 1996). Integral control creates an equilibrium point at which the tracking error is zero. In Khalil (2000) and Mahmoud and Khalil (1996),

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this (unknown) equilibrium point is stabilized using the following two-step design philosophy. First, robust control is designed to bring the trajectories to a small neighborhood of the equilibrium point. Within this neighborhood, the control acts as a high-gain feedback that stabilizes the equilibrium point. While Mahmoud and Khalil (1996) accomplishes this through continuous min–max control, Khalil (2000) uses continuous SMC (CSMC) to design the controller as a universal one, where the only precise information about the plant that is used is its relative degree and the sign of its high-frequency gain.

The asymptotic regulation achieved by integral action happens at the expense of degrading the transient performance. Even in the absence of control saturation, integral action makes the response more oscillatory. When the control saturates, integrator build-up results, causing large overshoots and settling times. We present a new approach for introducing integral action to alleviate this transient performance degradation. This is done within a CSMC design framework. The integrator is modified to provide integral action only inside the boundary layer, i.e., only "conditionally". The improvement in transient performance is shown analytically by proving that the output feedback CSMC recovers the performance of a state feedback ideal SMC. Preliminary results were presented in Sheshagiri and Khalil(2001,2002).

The rest of this paper is organized as follows. In Section 2, we motivate the key points of the design and results via an example. In Section 3, we describe the system under consideration, and state our assumptions and the control objective. The control design is presented in Section 4, while the analysis of the closed-loop system and performance recovery of an ideal SMC design are shown in Section 5. The specialization to the universal integral regulator design (Khalil, 2000) is done in Section 6. Finally, our conclusions are presented in Section 7.

2. Motivating example

Consider the second-order system

$$\begin{aligned}\dot{x}_1 &= x_2,\\ \dot{x}_2 &= ax_1^2 + bx_2 + cx_2^3 + u,\\ y &= x_1.\end{aligned} \tag{1}$$

The constants a, b and c are assumed to be unknown, but bounded with known bounds. The control objective is to regulate the output y to a constant value r. In ideal SMC design, the sliding surface can be chosen as $s = k_1 e_1 + e_2$, where $e_1 = y - r$, $e_2 = \dot{e}_1$, and $k_1 > 0$. This ensures that when motion is constrained to $s = 0$, the error e_1 converges asymptotically to zero. Differentiating, one obtains

$$\dot{s} = k_1 e_2 + a(e_1 + r)^2 + be_2 + ce_2^3 + u.$$

Finite-time convergence to, and invariance of, $s = 0$ can be achieved by choosing $u = u_1 + u_2$, where the equivalent control u_1 is designed to cancel known or nominal terms in the expression for $\dot{s}$ and can be taken as

$$u_1 = -k_1 e_2 - \hat{a}(e_1 + r)^2 - \hat{b}e_2 - \hat{c}e_2^3,$$

where $\hat{a}$, $\hat{b}$, and $\hat{c}$ are nominal values of a, b, and c, respectively. The switching control u_2 is designed to handle the uncertain terms in the resulting expression for $\dot{s}$ and can be taken as

$$u_2 = -[\alpha(e_1 + r)^2 + \beta|e_2| + \gamma|e_2|^3 + \delta]\,\mathrm{sgn}(s),$$

where the positive constants α, β, and γ are upper bounds on $|a - \hat{a}|$, $|b - \hat{b}|$, and $|c - \hat{c}|$, respectively, $\delta > 0$, and $\mathrm{sgn}(\cdot)$ is the signum function, defined by

$$\mathrm{sgn}(u) = \begin{cases} 1, & \text{if } u > 0,\\ -1, & \text{if } u < 0.\end{cases}$$

This choice ensures that s converges to zero in finite time and stays there for all future time, which guarantees that e_1 and e_2 converge to zero asymptotically. However, as is well-known, this design suffers from chattering in the presence of switching nonidealities or unmodeled high-frequency dynamics. Various approaches have been proposed to reduce or eliminate chattering, see, for example (Bartolini, Ferrara, Usai, & Utkin (2000); Levant & Fridman (2002)) and the references therein. The most common approach[1] is to replace the discontinuous term $\mathrm{sgn}(s)$ by its continuous approximation $\mathrm{sat}(s/\mu)$, where $\mathrm{sat}(\cdot)$ is the standard saturation function, defined by

$$\mathrm{sat}(u) = \begin{cases} u, & \text{if } |u| \leqslant 1,\\ \mathrm{sgn}(u), & \text{if } |u| > 1.\end{cases}$$

This method can eliminate chattering but often at the cost of a non-zero steady-state error, that is proportional to μ. In order to obtain smaller errors, it is therefore necessary to make μ smaller, which in turn, leads to chattering again.

It is possible to recover the asymptotic regulation achieved by ideal SMC by using integral control within a CSMC setting. Integral action is conventionally introduced by augmenting the system with an integrator driven by the tracking error, i.e., $\dot{\sigma} = e_1$. In the case of the particular example, suppose we do this and also modify the sliding surface to $s = k_0\sigma + k_1 e_1 + e_2$, where now k_0 and k_1 are chosen to ensure that the polynomial $\lambda^2 + k_1\lambda + k_0$ is Hurwitz, which guarantees that when motion is restricted to $s = 0$, the error e_1 converges asymptotically to zero. The previous steps can then be repeated to design u. In particular, we take

$$\begin{aligned}u_1 &= -k_0 e_1 - k_1 e_2 - \hat{a}(e_1 + r)^2 - \hat{b}e_2 - \hat{c}e_2^3,\\ u_2 &= -[\alpha(e_1 + r)^2 + \beta|e_2| + \gamma|e_2|^3 + \delta]\ \mathrm{sat}(s/\mu).\end{aligned}$$

[1] Another popular approach, discussed extensively in Bartolini et al. (2000) and Levant and Fridman (2002), is that of higher order sliding modes (HOSM), where the discontinuous control is relegated to higher-order derivatives of the input. While the presentation in Levant and Fridman (2002) is restricted to SISO systems, MIMO systems are considered in Bartolini et al. (2000), but under the assumption of full state feedback.

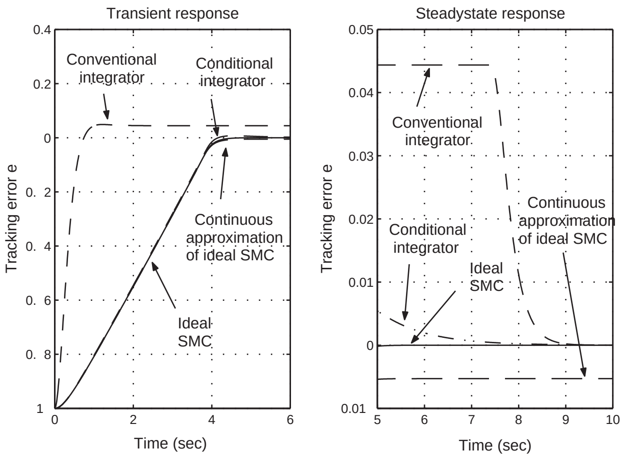

Fig. 1. Asymptotic error regulation with improved transient performance using the “conditional integrator”.

The presence of integral action guarantees that there is an equilibrium point, within $O(\mu)$ of the origin, at which $e_1 = 0$. Now, to achieve asymptotic regulation we do not need μ to be arbitrarily small; we only need it to be “small enough” to stabilize the equilibrium point.[2] However, while integral control, as designed above, can achieve asymptotic regulation, the transient response deteriorates when compared to that under ideal SMC.

To address the transient response degradation with conventional integral control, we modify the integrator design as follows. Let s be as before, i.e., $s = k_0\sigma + k_1 e_1 + e_2$, but now $k_0 > 0$ is arbitrary, $k_1 > 0$ is retained from the ideal SMC design, and σ is the output of

$$\dot{\sigma} = -k_0\sigma + \mu\,\mathrm{sat}(s/\mu). \qquad (2)$$

To see the relation of (2) to integral control, observe that inside the boundary layer $\{|s| \leqslant \mu\}$, (2) reduces to $\dot{\sigma} = k_1 e_1 + e_2 = k_1 e_1 + \dot{e}_1$, which implies that $e_1 = 0$ at equilibrium. Thus (2) represents a “conditional integrator” that provides integral action only inside the boundary layer. The control is taken as in the continuous approximation of ideal SMC, i.e., $u = u_1 + u_2$, where

$$u_1 = -k_1 e_2 - \hat{a}(e_1 + r)^2 - \hat{b}e_2 - \hat{c}e_2^3,$$
$$u_2 = -[\alpha(e_1 + r)^2 + \beta|e_2| + \gamma|e_2|^3 + \delta]\ \mathrm{sat}(s/\mu).$$

The simulation results are shown in Fig. 1. Numerical values used in the simulation are $a = 0.6$, $b = 2.5$, $c = 0.1$, $\hat{a} = 1$, $\hat{b} = 2$, $\hat{c} = 0$, $r = 1$, $\alpha = 0.5$, $\beta = 0.6$, $\gamma = 0.1$, $\delta = 1$, $\mu = 0.1$, and $x_1(0) = x_2(0) = \sigma(0) = 0$. The constant $k_1 = 5$ in the ideal SMC case, its continuous approximation, and the conditional integrator design, with $k_0 = 1$ in the conditional integrator design. The values of k_0 and k_1 in the conventional integrator design are taken as 25 and 10, respectively. The following observations can be made from Fig. 1: (i) the conventional integral control recovers the asymptotic regulation that is lost in the continuous approximation of ideal SMC (without integral control), but at the expense of degraded transient performance; in particular, the error convergence to zero is sluggish; (ii) the conditional integral design also achieves the task of asymptotic regulation, but without any degradation in transient performance; in fact, in the subplot for the transient behavior, the responses of the ideal SMC and the conditional integrator design are almost indistinguishable.

[2] We naturally expect this fact to be of consequence when there are switching nonidealities, and will show so through simulation later on.

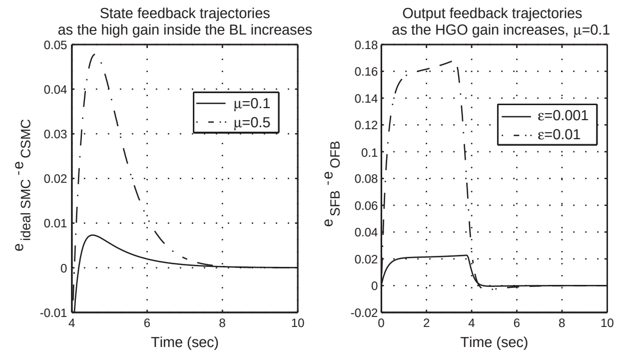

Fig. 2. Performance recovery under the output feedback conditional integrator design.

The transient performance recovery property of the conditional integrator design is also retained under output feedback, when the state e_2 is replaced by its estimate $\hat{e}_2$ obtained from the high-gain observer (HGO)

$$\dot{\hat{e}}_1 = \hat{e}_2 + \alpha_1(e_1 - \hat{e}_1)/\varepsilon,$$
$$\dot{\hat{e}}_2 = \alpha_2(e_1 - \hat{e}_1)/\varepsilon^2.$$

The positive constants α_1 and α_2 are chosen to assign the roots of the Hurwitz polynomial $\lambda^2 + \alpha_1\lambda + \alpha_2$, and ε is chosen sufficiently small. In order to take care of the peaking phenomenon associated with high-gain observers (Esfandiari & Khalil, 1992), the control is saturated outside a compact set of interest. Simulation results are shown in Fig. 2, with $\alpha_1 = 15$, $\alpha_2 = 50$, $\hat{e}_1(0) = \hat{e}_2(0) = 0$, and a saturation level of 50 for the control. Performance recovery is shown in two steps : (i) as μ tends to zero, the response under state feedback CSMC approaches ideal SMC, (ii) for fixed μ, the response under the output feedback CSMC approaches that under state feedback CSMC as ε tends to zero. For the general case, we will prove in Section 5 that the closed-loop trajectories under output feedback CSMC with the conditional integrator approach those of the state feedback ideal SMC as μ, ε tend to zero.

The previous discussion showed how the design of the controller proceeds in general. Since our design requires that the control be bounded, a possible simplification of the controller is to choose the equivalent control to be zero and the coefficient of the switching component to be constant, i.e.,

$$u = -k\ \mathrm{sat}(\hat{s}/\mu).$$

Since, in practical applications, a constraint on the control magnitude appears naturally as an actuator limit, one might simply choose k as the maximum permissible control magnitude. Since the only precise information about the plant that such a controller uses is its relative degree and the sign of its high-frequency gain, it is referred to as a *universal integral regulator* (Khalil, 2000). We will discuss the universal integral regulator design further in Section 6 and show that the integrator modification (2) can be interpreted, in this case, as a special choice of a traditional anti-windup scheme.

To continue with this discussion, note that the control magnitude required to accommodate a step change in r increases as the step increases. One way to deal with this when the control is constrained with an a priori specified bound is through trajectory planning schemes.[3] In order to illustrate this, we consider the following modification to the previous simulations. The control is replaced by $u = -k\ \mathrm{sat}(\hat{s}/\mu)$, with $k=50$. All other values are retained from the previous simulations, except r, which is increased to 1.5. One can verify, for example, by simulation, that the sliding condition is not satisfied. However, when the constant reference r is "smoothed" by passing it through the filter $1/(\tau s+1)^2$, with $\tau=0.5$, the control magnitude is now sufficient to overcome the uncertain terms and the sliding condition is satisfied. Furthermore, since $s(0)=0$, the sliding condition ensures that s stays inside the boundary layer for all future time, which along with $e_1(0)=0$ implies that the error e_1 itself is small for all time, under both the conventional as well as the conditional integrator designs. When the trajectory stays inside the boundary layer during the transient period, the conditional integrator acts as an integrator all the time; hence, we do not expect any significant difference between the transient responses of the two designs. The advantage of the conditional integrator design becomes clear when we consider an unexpected disturbance that causes an abrupt change in the state of the system. For example, consider an additive impulse-like disturbance $d(t)$ of magnitude 75 acting at the input of the system between $t=5$ and 5.1385 s. The response of the two designs are shown in Fig. 3. We see from the plot on the left that while the system responses are almost identical (indistinguishable in that plot) before the onset of the disturbance, the response to the disturbance is significantly degraded with the conventional integrator design. The response before the disturbance is seen better in the plot on the right.

Lastly, before we present the system description and the problem statement for the general case, we make a small digression. In order to highlight the issue of chattering, we repeat the first two simulations under the assumption that a time delay of $T=0.01$ s precedes the control input. The results are shown in Fig. 4.

We see from the figure that there is chattering in the control and that the property of asymptotic regulation is lost with

[3] A more detailed discussion on this issue can be found in the next section.

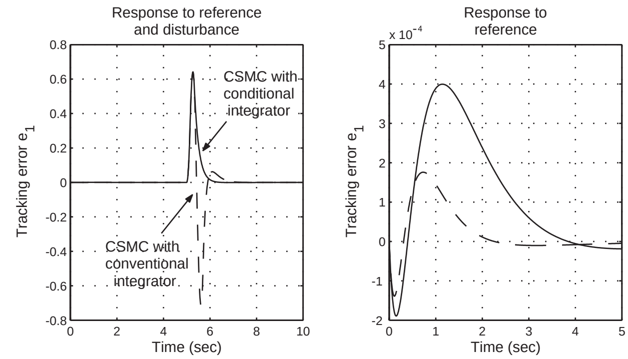

Fig. 3. Effect of disturbance on the conventional and the conditional integrator designs.

ideal SMC. Replacing the discontinuous control with its continuous approximation eliminates chattering when $\mu=0.1$, but at the expense of a relatively large non-zero steady-state error. Reducing μ to 0.01 results in chattering again. The non-zero steady-state error can be handled by the conditional integrator design, where as mentioned earlier, the value of μ does not have to be made arbitrarily small and hence we can expect that this design will not suffer from chattering. This is validated by the simulation results of Fig. 4. While we have included a simulation with a time delay to highlight a merit of this approach, we do not present any analysis for this case and shall not dwell on it further.

3. Problem statement

Consider an MIMO nonlinear system, modeled by

$$\dot{x} = f(x,\theta) + \sum_{i=1}^{m} g_i(x,\theta)[u_i + \delta_i(x,\theta,w)],$$
$$y_i = h_i(x,\theta), \quad 1 \leqslant i \leqslant m, \tag{3}$$

where $x \in R^n$ is the state, $u \in R^m$ is the control input, $y \in R^m$ is the output, θ is a vector of unknown constant parameters that belongs to a compact set $\Theta \subset R^p$, $w(t)$ is a piecewise continuous exogenous signal that belongs to a compact set $W \subset R^q$, $f(\cdot)$ and $g_i(\cdot)$ are smooth vector fields on $D \stackrel{\text{def}}{=} D_x \times \Theta$, where D_x is an open connected subset of R^n, $h_i(\cdot)$ are smooth functions on D, and the disturbances $\delta_i(\cdot)$ are continuous functions on $D \times W$. The formulation in (3) allows for matched disturbances that may depend on time-varying exogenous signals. We will specify a restriction on w shortly, when we are ready to state the control objective. Our first assumption is that the disturbance-free system (3) has a well-defined normal form, possibly with zero dynamics (Isidori, 1995).

Assumption 1. The system $\dot{x} = f(x,\theta) + g(x,\theta)u$, $y = h(x,\theta)$ has uniform vector relative degree $\{\rho_1, \rho_2, \ldots, \rho_m\}$

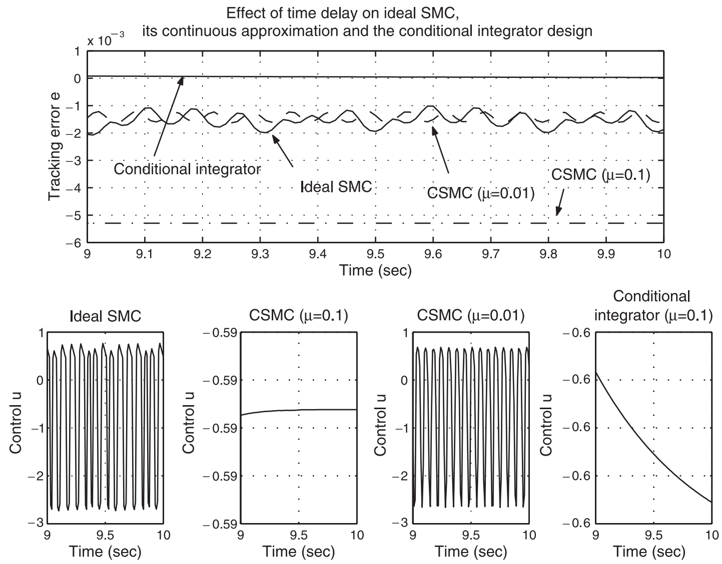

Fig. 4. Effect of time delay on the ideal SMC and the conditional integrator design.

in D_x, i.e.,

$$L_{g_j} L_f^k h_i(x) = 0 \quad \text{for } 0 \leqslant k \leqslant \rho_i - 2, 1 \leqslant i \leqslant m, 1 \leqslant j \leqslant m$$

and $A(x, \theta) \overset{\text{def}}{=} \{L_{g_j} L_f^{\rho_i - 1} h_i\}$ is nonsingular for all $x \in D_x$ and $\theta \in \Theta$. Furthermore, the distribution span$\{g_1, \ldots, g_m\}$ is involutive, and there is a change of variables

$$\begin{bmatrix} \eta \\ \xi \end{bmatrix} = T(x, \theta) = \begin{bmatrix} T_1(x, \theta) \\ T_2(x, \theta) \end{bmatrix}, \quad \eta \in R^{n-\rho}, \ \xi \in R^{\rho}, \qquad (4)$$

where $\xi = \{\xi^i\}$, with $\xi^i_j = L_f^{j-1} h_i$, $1 \leqslant j \leqslant \rho_i$, $1 \leqslant i \leqslant m$, and $\rho = \rho_1 + \rho_2 + \cdots + \rho_m$, such that $L_{g_j} \eta_i = 0 \, \forall 1 \leqslant j \leqslant m$, $1 \leqslant i \leqslant n - \rho$, and $T(x, \theta)$ is a diffeomorphism of D_x onto its image.

The vector relative degree and involutivity of the distribution span$\{g_1, \ldots, g_m\}$ guarantee the existence of the change of variables (4) locally, for each θ (Isidori, 1995). Assumption 1 goes beyond that by requiring (4) to hold on a given region, uniformly in θ.

With the change of variables (4), we rewrite (3) in the normal form

$$\begin{aligned} \dot{\eta} &= \phi(\eta, \xi, \theta), \\ \dot{\xi}^i &= A_i \xi^i + B_i \ [b_i(\eta, \xi, \theta) \\ &\quad + \sum_{j=1}^{m} a_{ij}(\eta, \xi, \theta)(u_j + \delta_j(\eta, \xi, \theta, w))], \end{aligned} \qquad (5)$$

where, for $1 \leqslant i \leqslant m$, the pair (A_i, B_i) is a controllable canonical form that represents a chain of ρ_i integrators, $b_i(\cdot) = L_f^{\rho_i} h_i$, and $\{a_{ij}(\cdot)\} = A(\cdot)$.

Our interest is in the regulation problem. To that end, we require that the exogenous signal $w(t)$ approaches a constant limit w_{ss}, i.e., $\lim_{t \to \infty} w(t) = w_{ss}$. In a similar vein, the reference $r_i(t)$ that the output y_i is required to asymptotically track has the following two properties:

- $r_i(t)$ and its derivatives up to the ρ_ith derivative are bounded, and $r_i^{(\rho_i)}(t)$ is piecewise continuous, for all $t \geqslant 0$,
- $\lim_{t \to \infty} r_i(t) = r_{iss}$ and $\lim_{t \to \infty} r_i^{(j)}(t) = 0$ for $1 \leqslant j \leqslant \rho_i$.

This class of signals includes constant signals as a special case. Formulating the problem with time-varying references, which are asymptotically constant, accommodates a common practice in many applications; for example, in "trajectory planning" schemes employed to achieve point-to-point motion in the control of robotic manipulators (Sciavicco & Siciliano, 1996, Chapter 5), or in pre-filter smoothing of a step command in the control of electric drives (Leonhard, 1985, Chapter 15). The formulation also takes advantage of the two-step process outlined in the introduction, in the following sense. When the reference satisfies the first property, the robust control design ensures ultimate boundedness of the tracking error. When the second property is satisfied as well, the integral action gaurantees that the error asymptotically converges to zero.

Let $r_{ss} = \{r_{iss}\}$, $\tilde{w}(t) = w - w_{ss}$, $v^i(t) = [r_i - r_{iss}, r_i^{(1)}, \ldots, r_i^{(\rho_i - 1)}]^{\mathrm{T}}$, $\varpi(t) = \{r_i^{(\rho_i)}\}$, and $v(t) = \{v^i\}$. By construction, $\tilde{w}(t)$, $v(t)$, and $\varpi(t)$ are bounded for all $t \geqslant 0$ and converge to zero as $t \to \infty$. Let $X \subset R^m$, $\Lambda \subset R^{\rho}$, and $\Lambda_0 \subset R^m$ be compact sets such that $r_{ss} \in X$, $v(t) \in \Lambda$, and $\varpi(t) \in \Lambda_0$ for all $t \geqslant 0$, and l_0 be a positive constant such that $\|v\| \leqslant l_0$ for all $v \in \Lambda$. Set $d = (r_{ss}, \theta, w_{ss})$ and $D_d = X \times \Theta \times W$. To solve the regulation problem, it is necessary that for every $d \in D_d$, thereexist an equilibrium point at which $y = r_{ss}$ and a control input that can maintain equilibrium. This is guaranteed by our next assumption.

Assumption 2. For each $d \in D_d$, there exist a unique equilibrium point $\bar{x} = \bar{x}(d) \in D_x$ and a unique control $\bar{u} = \bar{u}(d)$ such that $0 = f(\bar{x}, \theta) + g(\bar{x}, \theta)[\bar{u} + \delta(\bar{x}, \theta, w_{ss})]$ and $r_{ss} = h(\bar{x}, \theta)$.

With the change of variables (4), the equilibrium point $\bar{x}(d)$ maps into $(\bar{\eta}(d), \bar{\xi}(d))$, where $\bar{\xi}^i(d) = [r_{iss}, 0, \ldots, 0]^{\mathrm{T}}$. Let $z = \eta - \bar{\eta}$ and $e^i = \xi^i - \bar{\xi}^i - v^i$ and rewrite (5) as

$$\begin{aligned}
\dot{z} &= \phi(z, e+v, d),\\
\dot{e}^i &= A_i e^i + B_i\ [b_i(z, e+v, d) - r_i^{(\rho_i)}\\
&\quad + \sum_{j=1}^{m} a_{ij}(z, e+v, d)\\
&\quad \times (u_j + \delta_j(z, e+v, d, \tilde{w}))],
\end{aligned} \tag{6}$$

where, for convenience, we write the functions ϕ, b_i, a_{ij}, and δ_j in terms of the new variables. Since we do not necessarily require our assumptions to hold globally, we need to restrict our analysis in the (z, e) variables to a region that maps back into the domain D_x. The following assumption states such a restriction.

Assumption 3. There exist positive constants l_1 and l_2, independent of d, such that for all $d \in D_d$, $w \in W$, $v \in \Lambda$ and $\varpi \in \Lambda_0$,

$$e \in E \stackrel{\text{def}}{=} \{\|e\| < l_1\} \quad \text{and} \quad z \in Z \stackrel{\text{def}}{=} \{\|z\| < l_2\} \Rightarrow x \in D_x.$$

In the output feedback case, the only components of the state (z, e) that are available for feedback are $e_1^i = y_i - r_i$, $1 \leqslant i \leqslant m$. The unavailability of the partial-state e is dealt with by using a high-gain observer to estimate its unmeasured components. The unavailability of z is not an issue because we will design the control u to regulate the error e to zero and then rely on a minimum-phase-like assumption, stated below, to guarantee boundedness of z. The assumption has two parts. The first part states that with $(e + v)$ as the driving input, the system $\dot{z} = \phi(z, e+v, d)$ is input-to-state stable over a certain region (Khalil, 2002), which implies that with $e = 0$, the origin of $\dot{z} = \phi(z, 0, d)$ is asymptotically stable. This is strengthened in the second part of the assumption to local exponential stability of the origin.

Assumption 4.

(i) There exist a C^1 proper function $V_z : Z \to R_+$, possibly dependent on d, and class K functions $\lambda_i : [0, l_2) \to R_+ (i = 1, 2, 3)$ and $\gamma : [0, l_0 + l_1) \to R_+$, independent of d, such that

$$\lambda_1(\|z\|) \leqslant V_z(t, z, d) \leqslant \lambda_2(\|z\|),$$

$$\frac{\partial V_z}{\partial t} + \frac{\partial V_z}{\partial z}\phi(z, e+v, d) \leqslant -\lambda_3(\|z\|),$$
$$\forall\ \|z\| \geqslant \gamma(\|e+v\|)$$

for all $e \in E$, $z \in Z$, $v \in \Lambda$, and $d \in D$. Furthermore, $\gamma(l_0) < \lambda_2^{-1}(\lambda_1(l_2))$.

(ii) The equilibrium point $z = 0$ of $\dot{z} = \phi(z, 0, d)$ is exponentially stable, uniformly in d.

4. Controller design

Relying on the separation principle (Atassi & Khalil, 1999) that is common to the output feedback designs of Khalil (2000) and Mahmoud and Khalil (1996), we pursue the same procedure for designing the controller used in those papers. First, a globally bounded partial state-feedback controller that meets the design objectives is designed under the assumption that e is available for feedback. Next, a high-gain observer is used to estimate the derivatives of the measured outputs e_1^i.

4.1. Partial state feedback design

The first step in the sliding mode design is to specify a sliding surface on which sliding motion occurs (Young et al., 1999). In the absence of integral action, we define the sliding surface $s_i = 0$ by

$$s_i = \sum_{j=1}^{\rho_i - 1} k_j^i e_j^i + e_{\rho_i}^i, \tag{7}$$

where the positive constants $k_1^i, \ldots, k_{\rho_i-1}^i$ are chosen such that the polynomial

$$\lambda^{\rho_i - 1} + k_{\rho_i-1}^i \lambda^{\rho_i - 2} + \cdots + k_1^i$$

is Hurwitz, which guarantees that when motion is constrained to the surface $s_i = 0$, the tracking error e_1^i and its derivatives converge to zero. Differentiating (7) and using (6), we have

$$\dot{s}_i = F_i(z, e, v, d, r_i^{(\rho_i)}) + \sum_{j=1}^{m} a_{ij}(\cdot)[u_j + \delta_j(\cdot)], \tag{8}$$

where $F_i(\cdot) = b_i(\cdot) - r_i^{(\rho_i)} + \sum_{j=1}^{\rho_i-1} k_j^i e_{j+1}^i$. Let $F(z, e, v, d, \varpi) = \{F_i(z, e, v, d, r_i^{(\rho_i)})\}$. In the SISO case, (8) reduces to $\dot{s} = F(\cdot) + a(\cdot)[u + \delta(\cdot)]$ and a standard assumption in this case is to require the sign of $a(\cdot)$ to be known and $a(\cdot)$ to be bounded away from zero. Our next assumption can be thought of as a straightforward extension to the MIMO case.

Assumption 5. $A(z, e+v, d) = \Gamma(z, e+v, d)\hat{A}(e, v)$ where $\hat{A}$ is a known nonsingular matrix and $\Gamma = \mathrm{diag}[\gamma_1, \ldots, \gamma_m]$, with $\gamma_i(\cdot) \geqslant \gamma_0 > 0$, $1 \leqslant i \leqslant m$, for all $e \in E$, $z \in Z$, $v \in \Lambda$, $d \in D_d$, and some positive constant γ_0.

In the ideal SMC case, the control u can then be taken as

$$u = \hat{A}^{-1}(e, v)[-\hat{F}(e, v, \varpi) + v],$$
$$v_i = -\beta_i(e, v, \varpi)\ \mathrm{sgn}(s_i), \tag{9}$$

where $\hat{F}_i(\cdot)$ is a nominal value of $F_i(\cdot)$, which could be, but not restricted to, $\hat{F}_i(\cdot) = \sum_{j=1}^{\rho_i-1} k_i^j e_i^{j+1} - r_i^{(\rho_i)} + \hat{b}_i(\cdot)$, $\hat{b}_i(\cdot)$ being a nominal value of $b_i(\cdot)$, and the component v_i is designed to handle uncertainties. Note that $\hat{F}_i(\cdot) = 0$ is possible. The choice of $\beta_i(\cdot)$ will be made clear shortly.

Guided by the motivating example in Section 2, we introduce integral action as follows. First, the ideal sliding surface function s_i of (7) is modified to

$$s_i = k_0^i \sigma_i + \sum_{j=1}^{\rho_i-1} k_j^i e_j^i + e_{\rho_i}^i, \tag{10}$$

where σ_i is the output of

$$\dot{\sigma}_i = -k_0^i \sigma_i + \mu_i\ \mathrm{sat}\left(\frac{s_i}{\mu_i}\right),$$
$$\sigma_i(0) \in [-\mu_i/k_0^i, \mu_i/k_0^i], \tag{11}$$

with $k_0^i > 0$, and μ_i a small positive parameter to be specified later. Furthermore, as was done in Section 2, the ideal SMC (9) is modified to the continuous control

$$u = \hat{A}^{-1}(e, v)[-\hat{F}(e, v, \varpi) + v],$$
$$v_i = -\beta_i(e, v, \varpi)\ \mathrm{sat}(s_i/\mu_i). \tag{12}$$

Inside the boundary layer $\{|s_i| \leqslant \mu_i\}$, $\dot{\sigma}_i = \sum_{j=1}^{\rho_i-1} k_j^i e_j^i + e_{\rho_i}^i \stackrel{\text{def}}{=} e_a^i$, where the "augmented error" e_a^i is a linear combination of the tracking error e_1^i and its derivatives up to order $(\rho_i - 1)$. At equilibrium, $e_a^i = 0$, which implies that $e_1^i = 0$. Hence, Eq. (11) represents a conditional integrator, which provides integral action only inside the boundary layer. In the present case, the resulting equation for $\dot{s}_i$ can be written as

$$\dot{s}_i = \Delta_i(z, e, \varpi, \sigma, d, \tilde{w})$$
$$\qquad - \gamma_i(z, e + v, d)\beta_i(e, v, \varpi)\ \mathrm{sat}(s_i/\mu_i), \tag{13}$$

where $\Delta(\cdot) = \{\Delta_i(\cdot)\} = F(\cdot) - \Gamma(\cdot)\hat{F}(\cdot) + A(\cdot)\ \delta(\cdot) + \{k_0^i(-k_0^i\sigma_i + \mu_i\ \mathrm{sat}(s_i/\mu_i)\}$. In order to specify how $\beta_i(\cdot)$ is chosen, we make the following standard assumption.

Assumption 6. Let

$$\max\left|\frac{\Delta_i(\cdot)}{\gamma_i(\cdot)}\right| \leqslant \varrho_i(e, v, \varpi), \quad 1 \leqslant i \leqslant m \tag{14}$$

for some known functions $\varrho_i(\cdot)$, where the maximization is taken over all $(z, e, \sigma) \in \Psi_c$, $d \in D_d$, $v \in \Lambda$, $\varpi \in \Lambda_0$, and $w \in W$.

The compact set Ψ_c will be defined in the next section using Lyapunov functions, and will serve as an estimate of the region of attraction. The functions β_i are chosen as $\beta_i(\cdot) = \varrho_i(\cdot) + q_i$, where $q_i > 0$. From (13) and (14), it follows that inside Ψ_c, $s_i\dot{s}_i \leqslant -\gamma_0\ q_i\ |s_i|$, whenever $|s_i| \geqslant \mu_i$. We note that the right-hand side of (14) is independent of z and σ, even though Δ_i may depend on z and σ. The former, while restrictive, is necessiated by the fact that z is unavailable for feedback, and is justified since (14) is only required to hold over a compact set. The latter is done purely for convenience, and is not restrictive, since, as we shall see later on, $\|\sigma\| = O(\max_i\ \mu_i)$,so that the contribution of σ is not significant, provided the constants μ_i are sufficiently small.

4.2. Output feedback design

The output feedback design uses the following high-gain observer to estimate e^i:

$$\dot{\hat{e}}_j^i = \hat{e}_{j+1}^i + \alpha_j^i(e_1^i - \hat{e}_1^i)/(\varepsilon_i)^j, \quad 1 \leqslant j \leqslant \rho_i - 1,$$
$$\dot{\hat{e}}_{\rho_i}^i = \alpha_{\rho_i}^i(e_1^i - \hat{e}_1^i)/(\varepsilon_i)^{\rho_i}, \tag{15}$$

where $\varepsilon_i > 0$ is a design parameter, and the positive constants α_j^i are chosen such that the roots of $\lambda^{\rho_i} + \alpha_1^i\lambda^{\rho_i-1} + \cdots + \alpha_{\rho_i-1}^i\lambda + \alpha_{\rho_i}^i = 0$ have negative real parts. In (15), $\hat{e}_j^i$ is an estimate of e_j^i, the $(j-1)$th derivative of e_1^i. Let

$$\hat{s}_i = k_0^i\sigma_i + \sum_{j=1}^{\rho_i-1} k_j^i\hat{e}_j^i + \hat{e}_{\rho_i}^i \tag{16}$$

be the corresponding estimate of s_i,[4] where σ_i is now the output of

$$\dot{\sigma}_i = -k_0^i\sigma_i + \mu_i\ \mathrm{sat}(\hat{s}_i/\mu_i). \tag{17}$$

We replace e and s with their estimates $\hat{e}$ and $\hat{s}$ in the control (12), and saturate the control outside a compact set of interest. In particular, rewrite the control (12) as $u_i = \Upsilon_i(e, v, \varpi, \sigma)$ where $\Upsilon(\cdot) = \hat{A}^{-1}(\cdot)[-\hat{F}(\cdot) + v]$, with $v_i = -\beta_i(\cdot)\ \mathrm{sat}(s_i/\mu_i)$. Inside Ψ_c, e belongs to Λ_e, a compact subset of R^ρ. Let S_i be the maximum value of $|\Upsilon_i(e, v, \varpi, \sigma)|$, where the maximization is taken over all $v \in \Lambda$, $\varpi \in \Lambda_0$, $|\sigma_i| \leqslant \mu_i/k_0^i$ and $e \in \Lambda_{ee}$, where Λ_{ee} is a compact set that contains Λ_e in its interior. The control u is then taken as

$$u_i = S_i\ \mathrm{sat}(\Upsilon_i(\hat{e}, v, \varpi, \sigma)/S_i). \tag{18}$$

In summary, the output feedback controller is given by (15)–(18), where

$$\Upsilon(\hat{e}, v, \varpi, \sigma) = \hat{A}^{-1}(\hat{e}, v)[-\hat{F}(\hat{e}, v, \varpi) + v],$$
$$v_i = -\beta_i(\hat{e}, v, \varpi)\ \mathrm{sat}(\hat{s}_i/\mu_i),$$
$$\beta_i(\hat{e}, v, \varpi) = \varrho_i(\hat{e}, v, \varpi) + q_i. \tag{19}$$

[4] We can take $\hat{e}_1^i$ as the estimate provided by (15) or the measured output e_1^i.

To complete the controller design, we must specify how μ_i and ε_i are chosen. The parameters μ_i result from replacing an ideal SMC with its continuous approximation, and hence should be chosen "sufficiently small" to recover the performance of the ideal SMC. Similarly, in order for the output-feedback controller to recover the performance under state-feedback, the high-gain observer parameters ε_i should also be chosen "sufficiently small". Therefore, one might view μ_i and ε_i as tuning parameters and first reduce μ_i gradually until the transient response under partial state feedback is close enough to the ideal SMC, and then reduce ε_i gradually until the transient response under output feedback is close enough to that under state feedback. The asymptotic theory of the next section guarantees that this tuning procedure will work.

5. Closed-loop analysis

For $i=1,\ldots,m$, define $\zeta^i \in R^{\rho_i-1}$ by $(e^i)^{\mathrm{T}}=[(\zeta^i)^{\mathrm{T}}\ e^i_{\rho_i}]$ and write the closed-loop system in the standard singularly perturbed form

$$\begin{aligned}
\dot{\sigma}_i &= -k_0^i\sigma_i + \mu_i\ \mathrm{sat}((s_i - N_i(\varepsilon_i)\varphi^i)/\mu_i),\\
\dot{\zeta}^i &= M_i\zeta^i + C_i(s_i - k_0^i\sigma_i),\\
\dot{s}_i &= \Delta_i(z, e, \varpi, \sigma, d, \tilde{w})\\
&\quad - \gamma_i(z, e+v, d)\ \beta_i(e, v, \varpi)\ \mathrm{sat}(s_i/\mu_i) + \Delta_i^*(\cdot),\\
\dot{z} &= \phi(z, e+v, d),\\
\varepsilon_i\dot{\varphi}^i &= L_i\varphi^i + \varepsilon_i B_i \Bigg[b_i(\cdot) - r_i^{(\rho_i)}\\
&\quad + \sum_{j=1}^{m} a_{ij}(\cdot)(u_j + \delta_j(\cdot)) \Bigg],
\end{aligned} \tag{20}$$

where

$$M_i = \begin{bmatrix} 0 & 1 & \cdots & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \vdots & & & & \vdots\\ 0 & \cdots & \cdots & 0 & 1\\ -k_1^i & -k_2^i & \cdots & \cdots & -k^i_{\rho_i-1} \end{bmatrix}, \quad C_i = \begin{bmatrix} 0\\ 0\\ \vdots\\ 0\\ 1 \end{bmatrix},$$

$$L_i = \begin{bmatrix} -\alpha_1^i & 1 & \cdots & \cdots & 0\\ \vdots & 0 & 1 & \cdots & 0\\ \vdots & & & & \vdots\\ \vdots & \cdots & \cdots & 0 & 1\\ -\alpha^i_{\rho_i} & 0 & \cdots & \cdots & 0 \end{bmatrix},$$

$$N_i^{\mathrm{T}}(\varepsilon_i) = \begin{bmatrix} 0\\ k_i^2\varepsilon_i^{\rho_i-2}\\ \vdots\\ k_i^{\rho_i-1}\varepsilon_i \end{bmatrix},$$

$$\begin{aligned}
\Delta_i^*(\cdot) &= k_0^i\mu_i\ [\mathrm{sat}(\hat{s}_i/\mu_i) - \mathrm{sat}(s_i/\mu_i)]\\
&\quad + \sum_{j=1}^{m} a_{ij}(z, e+v, d)\\
&\quad \times [S_j\ \mathrm{sat}(\Upsilon_j(\hat{e}, v, \varpi, \sigma)/S_j) - \Upsilon_j(e, v, \varpi, \sigma)]
\end{aligned}$$

and the scaled estimation error $\varphi^i = \{\varphi^i_j\}$ is defined by

$$\varphi^i_j = (e^i_j - \hat{e}^i_j)/(\varepsilon_i)^{\rho_i - j}.$$

The matrices M_i and L_i are Hurwitz by design. Let $\mu=\{\mu_i\}$ and $\varepsilon=\{\varepsilon_i\}$.

5.1. Boundedness and convergence

The stability analysis shares many details with Khalil (2000) and Mahmoud and Khalil (1996).[5] The main difference between the present analysis and its counterparts in Khalil (2000) and Mahmoud and Khalil (1996) is treating σ_i and ζ^i separately, while in Khalil (2000) and Mahmoud and Khalil (1996), they are lumped together in one vector. As in Khalil (2000) and Mahmoud and Khalil (1996), it is both convenient as well as instructive to present the analysis in two parts. In the first part, we show that the controller parameters can be chosen to bring the trajectories to an arbitrarily small neighborhood of an equilibrium point, at which the tracking error is zero. In the second part, we show that the controller parameters can be further tuned to ensure asymptotic stabilization of this equilibrium point. To show the first part, we define appropriate Lyapunov functions for each of the five components of (20), i.e., s_i, σ_i, ζ^i, z, and φ^i, and use them to construct a compact set of interest $\Psi_c \times \Sigma_\varepsilon$ that serves as an estimate of the region of attraction. We show that this set is positively invariant for a suitable choice of the controller parameters and that trajectories starting in the interior of this set will eventually reach a "small" set $\Psi_\mu \times \Sigma_\varepsilon$ that shrinks to the origin as $\|\mu\|_\infty$ and $\|\varepsilon\|_\infty$ tend to zero. To that end, let $Q_i = Q_i^{\mathrm{T}} > 0$ and $P_i = P_i^{\mathrm{T}} > 0$ be the solutions of the Lyapunov equations $Q_iM_i + M_i^{\mathrm{T}}Q_i = -I$ and $P_iL_i + L_i^{\mathrm{T}}P_i = -I$, respectively. For the components s, σ, ζ, and φ, we use the quadratic Lyapunov functions

$$V_i^s(s_i) \stackrel{\text{def}}{=} \tfrac{1}{2}s_i^2, \quad V_i^\sigma(\sigma_i) \stackrel{\text{def}}{=} \tfrac{1}{2}\sigma_i^2, \quad V_i^\zeta(\zeta^i) \stackrel{\text{def}}{=} \zeta^{i\,\mathrm{T}} Q_i \zeta^i$$

and

$$V_i^\varphi(\varphi^i) \stackrel{\text{def}}{=} \varphi^{i\,\mathrm{T}} P_i \varphi^i,$$

respectively, and we use the Lyapunov function $V_z(t, z, d)$ for z. The sets Ψ_c and Σ_ε are defined by $\Psi_c \stackrel{\text{def}}{=} \Omega_c \times \Omega_{cz}$,

[5] We omit most of the details that are similar to Khalil (2000) and Mahmoud and Khalil (1996). A complete account of such details can be found in Seshagiri (2003).

$\Omega_c \stackrel{\text{def}}{=} (\prod_{i=1}^m \Omega_{c_i})$, $\Sigma_\varepsilon \stackrel{\text{def}}{=} \prod_{i=1}^m \Sigma_{\varepsilon_i}$, where

$$\Omega_{c_i} \stackrel{\text{def}}{=} \{V_i(\zeta^i) \leqslant (c_i+\mu_i)^2\chi_i,\ \ V_i^s(s_i) \leqslant \tfrac{1}{2}c_i^2,\ V_i^\sigma(\sigma_i) \leqslant \tfrac{1}{2}(\mu_i/k_0^i)^2\},$$
$$\Omega_{cz} \stackrel{\text{def}}{=} \{V_z(t,z,d) \leqslant \lambda_4(l_0 + l_3\|c\|)\},$$
$$\Sigma_{\varepsilon_i} \stackrel{\text{def}}{=} \{V_i^\varphi(\varphi^i) \leqslant \varepsilon_i^2 \vartheta_i\}. \tag{21}$$

$c_i > \mu_i$ is a positive constant, $\lambda_4 = \lambda_2 \circ \gamma$ is a class K function, and χ_i, l_3, and ϑ_i are positive constants independent of μ_i and ε_i to be specified shortly.

Before we show that $\Psi_c \times \Sigma_\varepsilon$ serves as an estimate of the region of attraction, we need to ensure that $(z,e,\sigma) \in \Psi_c$ implies that $(z,e) \in Z \times E$. It can be verified that $\|e\| \leqslant l_3\|c\|$ in Ω_c, where l_3 is a positive constant independent of c. Using this fact, along with Assumption 4(i), it follows that choosing c to ensure that $l_3\|c\| < \min\{l_1, \lambda_4^{-1}(\lambda_1(l_2)) - l_0\}$ guarantees that $(z,e) \in Z \times E$ for all $(z,e,\sigma) \in \Psi_c$.

Since the boundaries of the set $\Psi_c \times \Sigma_\varepsilon$ are formed of Lyapunov surfaces, to show that this set is positively invariant, it suffices to show that the derivatives of the corresponding Lyapunov functions are non-positive on the respective boundaries. Using the fact that $|s_i| \leqslant c_i$, $|\sigma_i| \leqslant \mu_i/k_0^i$ in Ψ_c, and the inequality

$$\dot V_i^\zeta \leqslant -\|\zeta^i\|^2 + 2\|\zeta^i\|\ \|Q_i C_i\|\ (|s_i| + k_0^i|\sigma_i|),$$

it is easy to show that $\dot V_i^\zeta \leqslant 0$ on the boundary $V_i = (c_i+\mu_i)^2\chi_i$ for the choice $\chi_i = 4\|Q_iC_i\|^2\lambda_{\max}(Q_i)$. Since $\sigma_i\dot\sigma_i \leqslant -k_0^i|\sigma_i|^2 + \mu_i|\sigma_i|$, it follows that $\dot V_i^\sigma \leqslant 0$ on the boundary $V_i^\sigma = \frac{1}{2}\left(\mu_i/k_0^i\right)^2$. Next, we consider the $\dot s_i$ equation, which differs from (13) only in the term Δ_i^*. Inside Σ_{ε_i}, $\|\varphi^i\| = O(\varepsilon_i)$, which can be used to show that, for sufficiently small ε_i, the control is not saturated inside $\Psi_c \times \Sigma_\varepsilon$.Using this, along with $s_i = \hat s_i + N_i(\varepsilon_i)\varphi^i$, it can be shown that Δ_i^* is $O(\|\varepsilon\|_\infty)$ inside $\Psi_c \times \Sigma_\varepsilon$. Let ε_i be small enough that $|\Delta_i^*(\cdot)| < \gamma_0 q_i$. On the boundary $V_i^s = \frac{1}{2}c_i^2$ we have $\mathrm{sat}(s_i/\mu_i) = \mathrm{sgn}(s_i)$, so that

$$\dot V_i^s \leqslant -|s_i|[\gamma_i(\cdot)\beta_i(\cdot) - |\Delta_i(\cdot)| - |\Delta_i^*(\cdot)|].$$

Using (14), the definition of β_i, and the fact that $|\Delta_i^*(\cdot)| < \gamma_0 q_i$, it follows that $\dot V_i^s < 0$ on the boundary $V_i^s = \frac{1}{2}c_i^2$. Assumption 4(i) shows that $\dot V_z \leqslant 0$ on the boundary $V_z = \lambda_4(l_0 + l_3\|c\|)$. Finally, using the inequality

$$\dot V_i^\varphi \leqslant -\frac{\|\varphi^i\|^2}{\varepsilon_i} + 2\ \|\varphi^i\|\ \|M_iB_i\|\ \Xi_i,$$

where $\Xi_i = \max|b_i(\cdot) - r_i^{(\rho_i)} + \sum_{j=1}^m a_{ij}(\cdot)[u_j + \delta_j(\cdot)]|$, with the maximization taken over all $(z,e,\sigma) \in \Psi_c$, $d \in D_d$, $v \in \Lambda$, $\varpi \in \Lambda_0$, $w \in W$, and $\varphi \in \Sigma_\varepsilon$, it follows that $\dot V_i^\varphi \leqslant 0$ on the boundary $V_i^\varphi = \varepsilon_i^2\vartheta_i$ for the choice $\vartheta_i > 4\ \|M_iB_i\|^2\ \lambda_{\max}(M_i)\ \Xi_i^2$. Hence, $\Psi_c \times \Sigma_\varepsilon$ is positively invariant.

Our next step is to show that for any bounded $\hat e(0)$, and any $(z(0), e(0), \sigma(0)) \in \Omega_b$, where $0 < b_i < c_i$, it is possible to choose ε_i such that the trajectory enters the set $\Psi_c \times \Sigma_\varepsilon$ in finite time. Since, for all $(z,e,\sigma) \in \Omega_c$, the right-hand side of the slow equation of (20) is bounded uniformly in ε, for all $(z(0), e(0), \sigma(0)) \in \Omega_b$ there is a finite time T_0, independent of ε, such that for all $0 \leqslant t \leqslant T_0$, $(z(t), e(t), \sigma(t)) \in \Omega_c$. During this interval, we have

$$\dot V_i^\varphi \leqslant -\alpha_\varphi^i\|\varphi^i\|^2 \quad \text{for } V_i^\varphi(\varphi^i) \geqslant \varepsilon_i^2\vartheta_i$$

for some positive constant α_φ^i. This inequality can be used to show that $\varphi^i(t)$ enters Σ_{ε_i} within the time interval $[0, T(\varepsilon)]$, where $\lim_{\varepsilon\to 0}T(\varepsilon) = 0$ (Atassi & Khalil, 1999). Therefore, by choosing ε_i small enough we can ensure that $T(\varepsilon) < T_0$.

The argument that the set $\Psi_c \times \Sigma_\varepsilon$ is positively invariant can be extended to show that trajectories starting inside it reach the set $\Psi_\mu \times \Sigma_\varepsilon$ in finite time, where

$$\Psi_\mu \stackrel{\text{def}}{=} \Omega_\mu \times \{V_z(t,z,d) \leqslant \lambda_4(\|\mu\|_\infty r^*)\},$$
$$\Omega_\mu \stackrel{\text{def}}{=} \prod_{i=1}^m \{(e^i, \sigma_i) : |s_i| \leqslant \mu_i(1-\varsigma_i),\ |\sigma_i| \leqslant \frac{\mu_i}{k_0^i},\ \ V_i^\zeta(\zeta^i) \leqslant 16\mu_i^2\chi_i\},$$
$$\Sigma_\varepsilon \stackrel{\text{def}}{=} \prod_{i=1}^m \{\varphi^i \in R^{\rho_i} : V_i^\varphi(\varphi^i) \leqslant \|\varepsilon\|_\infty^2\vartheta_i\}. \tag{22}$$

$0 < \varsigma_i < 1$ is chosen such that $\max\ \varrho_i \leqslant q_i/(2\varsigma_i) - q_i$, ε_i is small enough that $|N_i(\varepsilon_i)\varphi^i| < \varsigma_i\ \mu_i$, and $r^* > \alpha^*$, where α^* is a positive constant such that $\|e\| \leqslant \|\mu\|_\infty\alpha^*$ for all $e \in \Omega_\mu$. An argument similar to the one for $\Psi_c \times \Sigma_\varepsilon$ can be used to show that $\Psi_\mu \times \Sigma_\varepsilon$ is positively invariant. This completes the first part of the analysis.

To prove the second part, note that when $\tilde w = 0$, $v = 0$ and $\varpi = 0$, the system has a unique equilibrium point ($z = 0, e = 0, \sigma_i = \bar\sigma_i, \varphi = 0$). Let $\bar s_i = k_0^i\bar\sigma_i$ be the corresponding equilibrium value of s_i, $\tilde\sigma = \sigma - \bar\sigma$, and $\tilde s = s - \bar s$. By the converse Lyapunov theorem (Khalil, 2002), Assumption 4(ii) implies that in some neighborhood of $z = 0$ there is a Lyapunov function $W_z(z,d)$ that satisfies

$$\lambda_5\|z\|^2 \leqslant W_z \leqslant \lambda_6\|z\|^2, \quad (\partial W_z/\partial z)\phi(z,0,d) \leqslant -\lambda_7\|z\|^2,$$

and

$$\left\|\partial W_z/\partial z\right\| \leqslant \lambda_8\|z\|$$

for some positive constants λ_5 to λ_8, independent of d. Let Q=blockdiag$[Q_1, \ldots, Q_m]$, and P=blockdiag$[P_1, \ldots, P_m]$. Using

$$V = W_z(z,d) + \lambda_9\zeta^{\mathrm T}Q\zeta + \tfrac{1}{2}\lambda_{10}\|\tilde\sigma\|^2 + \tfrac{1}{2}\|\tilde s\|^2 + \varphi^{\mathrm T}P\varphi$$

as a Lyapunov function candidate, where $\lambda_9, \lambda_{10} > 0$, it can be verified that, by first taking λ_9 large enough, then λ_{10} large enough, then $\|\mu\|_\infty$ small enough, and lastly $\|\varepsilon\|_\infty$ small enough, $\dot V$ satisfies an inequality of the form

$$\dot V \leqslant -\lambda_{11}V + \lambda_{12}\sqrt{V}(\|v(t)\| + \|\varpi(t)\| + \|\tilde w(t)\|) \tag{23}$$

for some positive constants λ_{11} and λ_{12}, uniformly in μ and ε. Since $\tilde{w}$, $v(t)$, $\varpi(t) \to 0$ as $t \to \infty$, the preceding inequality can be used to show that all trajectories approach the equilibrium point ($z=0$, $e=0$, $\sigma=\bar{\sigma}$, $\varphi=0$) as t tends to infinity. If all assumptions hold globally, the controller can achieve semiglobal regulation. We summarize our conclusions in the following theorem.

Theorem 1. *Suppose Assumptions* 1 *through* 6 *are satisfied, the constants c_i, χ_i, ϑ_i, and l_3 are chosen as described before, $\hat{e}(0)$ is bounded, and the initial states $(z(0), e(0), \sigma(0))$ belong to the set Ψ_b, where $0 < b_i < c_i$. Then, there exists $\mu^* > 0$, and for each μ with $\|\mu\|_\infty \in (0, \mu^*]$, there exists $\varepsilon^* = \varepsilon^*(\mu) > 0$, such that, for $\mu_i \in (0, \mu^*]$ and $\varepsilon_i \in (0, \varepsilon^*]$, all state variables of the closed-loop system under the output feedback controller* (15)–(19) *are bounded, and $\lim_{t\to\infty} e(t) = 0$. If, in addition, all the assumptions hold globally, then, given compact sets $N \subset R^n$ and $M \subset R^\rho$, the foregoing conclusion holds for all $(z(0), e(0)) \in N$ and $\hat{e}(0) \in M$, provided Ψ_c is chosen large enough to include N.*

5.2. Performance

We saw in Section 2, via simulation, that the output feedback CSMC with a conditional integrator recovers the performance of the state-feedback ideal SMC. The following theorem shows that the closed-loop trajectories under the two controllers can be made arbitrarily close.

Theorem 2. *Let $X = (z, e)$ be part of the state of the closed-loop system for* (6) *under the output feedback CSMC* (15)–(19), *and $X^* = (z^*, e^*)$ be the state of the closed-loop system under the state feedback ideal SMC control* (7) *and* (9), *with $X(0) = X^*(0)$. Then, under the hypotheses of Theorem* 1, *for every $\tau > 0$, there exists $\mu^* > 0$, and for each μ with $\|\mu\|_\infty \in (0, \mu^*]$, there exists $\varepsilon^* = \varepsilon^*(\mu) > 0$, such that, for $\mu_i \in (0, \mu^*]$ and $\varepsilon_i \in (0, \varepsilon^*]$, $\|X(t) - X^*(t)\| \leqslant \tau\ \forall\ t \geqslant 0$.*

Proof. We prove the theorem in two parts. First, we look at the trajectories under state feedback CSMC with the conditional integrator. Let $X^\dagger = (z^\dagger, e^\dagger)$ be part of the state of the closed-loop system under the control (10)–(12), with $X^\dagger(0) = X^*(0)$. For this case, we show that, for sufficiently small μ_i, $X^\dagger(t) - X^*(t) = O(\|\mu\|_\infty)\ \forall\ t \geqslant 0$. Let $s^\dagger$ and s^* be the corresponding sliding surface functions of the two systems. Let $I_M = \{1, \ldots, m\}$ and $t_1 = \min\{t_1^\dagger, t_1^*\}$, where

$$t_1^\dagger = \min_{i\in I_M} \{t : |s_i^\dagger(t)| \leqslant \mu_i\} \text{ and}$$
$$t_1^* = \min_{i\in I_M} \{t : |s_i^*(t)| = 0\}.$$

If $t_1 > 0$, using $\mathrm{sat}(s_i^\dagger(t)/\mu_i) = \mathrm{sgn}(s_i^*(t))\ \forall\ 0 \leqslant t < t_1$, it can be shown that $X^\dagger(t) = X^*(t)\ \forall\ 0 \leqslant t \leqslant t_1$. Next, we consider $X^\dagger(t)$ and $X^*(t)$ in the time interval $t \geqslant t_1$. Let

$$I_1 = \{i : |s_i^\dagger(t_1)| \leqslant \mu_i\} \cup \{i : s_i^*(t_1) = 0\}.$$

Since $X^\dagger(t_1) = X^*(t_1)$, $|s_i^\dagger(t_1) - s_i^*(t_1)| = |k_0^i\ \sigma_i^\dagger(t_1)| \leqslant \mu_i$. Using this, along with the definition of I_1 and the fact that $|s_i^\dagger(t)|$ and $|s_i^*(t)|$ monotonically converge to the positively invariant sets $\{|s_i^\dagger| \leqslant \mu_i\}$ and $\{0\}$, respectively, it can be shown that for all $i \in I_1$, $|s_i^\dagger(t) - s_i^*(t)| \leqslant 3\mu_i$ for all $t \geqslant t_1$. It follows that for all $i \in I_1$, $s_i^\dagger(t) - s_i^*(t) = O(\mu_i)\ \forall\ t \geqslant 0$. Since the equations for $\zeta^{i\dagger}$ and ζ^{i*} are identical stable linear equations, driven by inputs $s_i^\dagger - k_0^i\ \sigma_i^\dagger$ ands_i^* respectively, where $|k_0^i\ \sigma_i^\dagger| \leqslant \mu_i$ and $s_i^\dagger - s_i^* = O(\mu_i)$, continuity of solutions on the infinite time interval (Khalil, 2002, Theorem 9.1) can be used to show that for sufficiently small μ_i, $\zeta^{i\dagger}(t) - \zeta^{i*}(t) = O(\mu_i)$ and hence $e^{i\dagger}(t) - e^{i*}(t) = O(\mu_i)$ for all $i \in I_1$ and $t \geqslant t_1$. In particular, if $I_1 = I_M$, then $e^\dagger(t) - e^*(t) = O(\|\mu\|_\infty)$ for all $t \geqslant t_1$, which can then be used to show that $z^\dagger(t) - z^*(t) = O(\|\mu\|_\infty)$ for all $t \geqslant t_1$, so that the result follows.

If $I_1 \neq I_M$, let $t_2 \in (t_1, \infty) = \min\{t_2^\dagger, t_2^*)$, where

$$t_2^\dagger = \min_{i\in I_M\setminus I_1} \{t : |s_i^\dagger(t)| \leqslant \mu_i\} \quad \text{and}$$
$$t_2^* = \min_{i\in I_M\setminus I_1} \{t : |s_i^*(t)| = 0\}.$$

Let $X_1^\dagger$ be the part of the state $X^\dagger$ with the components e^i, $i \in I_1$, deleted and X_1^* be the corresponding part of X^*. For $i \in I_M \setminus I_1$, we have $\mathrm{sat}(s_i^\dagger/\mu_i) = \mathrm{sgn}(s_i^*) = \mathrm{sign}(s_i^\dagger)\ \forall\ t_1 \leqslant t < t_2$, so that, during this period, the right-hand side of the equations for $X_1^\dagger$ and X_1^* are Lipschitz functions of their arguments. Viewing $X_1^\dagger$ and X_1^* as states of systems driven by inputs $(\sigma_i^\dagger, e^{i\dagger})$ and e^{i*} respectively, $i \in I_1$, and using the fact that $|k_0^i\ \sigma_i^\dagger| \leqslant \mu_i$, and $e^{i\dagger} - e^{i*} = O(\mu_i)$, the results of (Khalil, 2002, Theorem 3.4), dealing with continuity of solutions on compact time-intervals, can be used to show that, for sufficiently small μ_i, $X_1^\dagger(t) - X_1^*(t) = O(\|\mu\|_\infty)\ \forall\ t_1 \leqslant t \leqslant t_2$.Using this, the previous arguments involving (Khalil, 2002, Theorem 9.1) can then be repeated to show that, for sufficiently small μ_i, $e^{i\dagger}(t) - e^{i*}(t) = O(\mu_i)\ \forall\ t \geqslant t_2$ and all $i \in I_2$, where

$$I_2 = \{i \in I_M \setminus I_1 : s_i^\dagger(t_2)| \leqslant \mu_i\}$$
$$\times \cup \{i \in I_M \setminus I_1 : s_i^*(t_2) = 0\}.$$

In particular, if $I_1 \cup I_2 = I_M$, then $e^\dagger(t) - e^*(t) = O(\|\mu\|_\infty)\ \forall\ t \geqslant t_2$, which can then be used to show that $z^\dagger(t) - z^*(t) = O(\|\mu\|_\infty)\ \forall\ t \geqslant t_2$, so that the result follows. If $I_1 \cup I_2 \neq I_M$, the result follows by an inductive argument that uses (Khalil, 2002, Theorem 3.4) and (Khalil, 2002, Theorem 9.1) alternately. In particular, this completes the first part of the proof, which shows that there exists $\mu^* > 0$ such that $\|\mu\|_\infty \in (0, \mu^*] \Rightarrow \|X^\dagger(t) - X^*(t)\| \leqslant \tau/2\ \forall\ t \geqslant 0$.

In the second part of the proof, we use the idea in Atassi and Khalil (1999) to show that the trajectories X

of the system under output feedback approach the trajectories $X^\dagger$ under state feedback as $\varepsilon \to 0$. In particular, we show that there exists $\varepsilon^* = \varepsilon^*(\mu)$ such that for all $\varepsilon_i \leqslant \varepsilon^*$, $\|X(t) - X^\dagger(t)\| \leqslant \tau/2\ \forall\ t \geqslant 0$. This is done by dividing the time interval $[0, \infty)$ into three sub-intervals $[0, T(\varepsilon)]$, $[T(\varepsilon), T_3]$ and $[T_3, \infty)$ and showing that the inequality $\|X(t) - X^\dagger(t)\| \leqslant \tau/2$ holds over each of these sub-intervals. From asymptotic stability of the two systems, we know that there exists a finite time T_3, independent of ε, such that $\|X(t) - X^\dagger(t)\| \leqslant \tau/2\ \forall\ t \geqslant T_3$. Also, as mentioned in Section 5.1, there is a time interval $[0, T(\varepsilon)]$, with $T(\varepsilon) \to 0$ as $\varepsilon \to 0$, during which the fast variable φ decays to an $O(\|\varepsilon\|_\infty)$ value. It can be shown that global boundedness of the controls implies that over this interval, $\|X(t) - X^\dagger(t)\| \leqslant \lambda_0 T(\varepsilon)$,for some positive constant λ_0 that is independent of ε. Since $T(\varepsilon) \to 0$ as $\varepsilon \to 0$, for small enough $\|\varepsilon\|_\infty$, $\|X(t) - X^\dagger(t)\| \leqslant \tau/2\ \forall\ t \in [0, T(\varepsilon)]$. Lastly, noting that $X(T(\varepsilon)) - X^\dagger(T(\varepsilon)) \to 0$ as $\varepsilon \to 0$ and φ is $O(\|\varepsilon\|_\infty)$, and using the continuous dependence of the solutions of differential equations on compact time intervals (Khalil, 2002, Theorem 3.4), one can show that it is possible to choose ε to satisfy the inequality $\|X(t) - X^\dagger(t)\| \leqslant \tau/2$ over the time interval $[T(\varepsilon), T_3]$. This shows that $\|X(t) - X^\dagger(t)\| \leqslant \tau/2\ \forall\ t \geqslant 0$. The result then follows from the triangle inequality.

6. Universal integral regulator

For SISO systems, the flexibility that is available in the choice of the functions $\hat{F}$ and β can be exploited to simplify the controller to

$$\begin{aligned} u &= -k\ \mathrm{sat}(\hat{s}/\mu) \\ &= -k\ \mathrm{sat}\left(\frac{k_0\sigma + k_1 e_1 + k_2\hat{e}_2 + \cdots + \hat{e}_\rho}{\mu}\right). \end{aligned} \tag{24}$$

As mentioned in Section 2, this particular design, while having a simple structure, is also natural since the control is required to be bounded. It is clear from (24) that the only precise knowledge about the plant that is used is its relative degree and the sign of its high-frequency gain $L_g L_f^{\rho-1} h$. This "universal" design was first presented in Khalil (2000), for the conventional integrator, where it was shown that the structure of the universal integral regulator coincides with the classical PI and PID controllers, followed by saturation, for relative degree one and two plants, respectively.

In the present case, when $\rho = 1$, the integrator equation can be rewritten as $\dot{\sigma} = e_1 + (\mu/k)(\hat{u} - u)$, where

$$u = -k\ \mathrm{sat}\left(\frac{k_0\sigma + e_1}{\mu}\right) \quad \text{and} \quad \hat{u} = -k\left(\frac{k_0\sigma + e_1}{\mu}\right).$$

The term $\hat{u}$ is the "unsaturated version" of the control u, so that the controller (24) has the structure shown in Fig. 5. It is a PI controller with "anti-windup" (Fertik & Ross, 1967), followed by saturation.

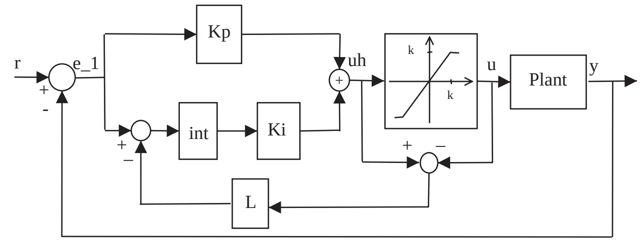

Fig. 5. Universal regulator for relative degree one systems : PI controller with anti-windup, followed by saturation; $K_I = kk_0/\mu$, $K_P = k/\mu$, and $L = \mu/k$.

In the relative degree ρ case, the integrator equation can be rewritten as $\dot{\sigma} = e_a + (\mu/k)(\hat{u} - u)$, where $e_a = \sum_{j=1}^{\rho-1} k_j e_j + e_\rho$ is the augmented error that was defined in Section 4. It is clear from the expression for $\dot{\sigma}$ that the anti-windup structure of Fig. 5 is retained in this case as well. The control (24) now represents a "PID$^{\rho-1}$ controller", with a conditional (anti-windup) integrator, followed by saturation. This interpretation of the conditional integrator as a specially tuned version of an anti-windup scheme for the universal integral regulator design was presented in a preliminary version of this paper (Seshagiri & Khalil, 2001).

7. Conclusion

We have presented a new approach to introducing integral action in the control of nonlinear systems, which captures the regional and semi-global asymptotic regulation results of Khalil (2000) and Mahmoud and Khalil (1996), while improving the transient response. In the new approach, the integrator is designed in such a way that it provides integral action only "conditionally", effectively eliminating the performance degradation. The improvement in performance is demonstrated analytically by showing that the output-feedback continuous sliding mode controller, with conditional integrator, recovers the performance of an ideal state-feedback sliding mode controller, without integral action, as the controller parameters μ_i and ε_i tend to zero. In view of this result, the control design can start with the ideal state-feedback sliding-mode control, where the parameters of the sliding surface $s_i^* = 0$ are chosen to meet the transient response specifications. Then, integral action is introduced by modifying s_i^* to $s_i = k_i^0 \sigma_i + s_i^*$, with $\dot{\sigma}_i = -k_i^0 \sigma_i + \mu_i\ \mathrm{sat}(s_i/\mu_i)$. The discontinuous term $\mathrm{sgn}(s_i^*)$ in the ideal SMC is replaced by $\mathrm{sat}(s_i/\mu_i)$. The parameters μ_i are reduced gradually until the transient response is close enough to the ideal case. Finally, a high-gain observer is brought in to estimate the derivatives of the tracking error. The observer parameters ε_i are gradually reduced until the transient performance is close enough to the ideal case. Note that in the ideal SMC design, the inequality that corresponds to (14) will have Δ_i terms that do not account for the σ_i variables. However, since σ_i is $O(\mu_i)$, theβ_i's of the ideal SMC design will still work in the presence of the conditional integrator, provided the μ_i's are sufficiently small.

While modifying the integral control designs of Khalil (2000) and Mahmoud and Khalil (1996) from conventional to conditional integrators, we have also extended the problem statement to MIMO systems and allowed time-varying matched disturbances. Moreover, we proved that the trajectories under output feedback approach those under state feedback as $\varepsilon \to 0$. This property also holds for Khalil (2000) and Mahmoud and Khalil (1996), but was not proved there.

Further work is needed in understanding how to fine-tune the controller parameters. For example, it is not hard to see that, when the level of the control is fixed a priori, there is a trade-off between the region of attraction and the speed of convergence, which is dictated by the choice of the sliding-surface parameters. Identifying such trade-offs will give insight into how the parameters can be tuned to achieve specific objectives, and also identify possible limitations on the achievable performance.

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