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Output Feedback Control of Nonlinear Systems Using RBF Neural Networks

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Abstract

An adaptive output feedback control scheme is presented for output tracking of a class of continuous-time nonlinear plants. An RBF neural network is used to adaptively compensate for the plant nonlinearities. The network weights are adapted using a Lyapunov-based design. The method uses parameter projection, control saturation, and a high-gain observer to achieve semi-global uniform ultimate boundedness. The efficacy of the proposed method is demonstrated through simulations. The simulations also show that by using adaptive control in conjunction with robust control, it is possible to tolerate larger approximation errors resulting from the use of lower-order networks.

1 Introduction

In recent years, the analytical study of adaptive nonlinear control systems using universal function approximators has received much attention. Typically, these methods use neural networks as approximation models for the unknown system nonlinearities [2, 3, 5, 6, 7]. A key assumption in most of these methods is that all the states of the plant are available for feedback. In [1], Aloliwi and Khalil developed an adaptive output feedback controller for a class of nonlinear systems and pointed out the potential application of their method to linear-in-the-weight neural networks. In this paper, we investigate the use of a radial basis function (RBF) neural network for the purpose.

2 Problem Statement

The system under consideration is represented by the following model(see [1]), with v being the control input and y the measured output:

$$\left.\begin{array}{rcl} \dot{x}_i &=& x_{i+1}, \quad 1 \le i \le n-1 \\ \dot{x}_n &=& F(x,z) + G(x,z)v \\ \dot{z}_i &=& z_{i+1}, \quad 1 \le i \le m-1 \\ \dot{z}_m &=& v \\ y &=& x_1 \end{array}\right\} \qquad (1)$$

where $x = [x_1, \ldots, x_n]^T$, $z = [z_1, \ldots, z_m]^T$.

Assumption 1 $|G(x,z)| \ge k_1 > 0 \ \forall \ x \in R^n$ and $z \in R^m$.

Assumption 1 ensures the existence a global diffeomorphism,

$$\begin{bmatrix} x \\ \zeta \end{bmatrix} = \begin{bmatrix} x \\ T_1(x,z) \end{bmatrix} \stackrel{\text{def}}{=} T(x,z)$$

with $T_1(0,0) = 0$, which transforms the last m state equations of (1) into $\dot{\zeta} = H(\zeta, x)$. This, together with the first n state equations of (1), defines a global normal form. The objective is to design an output feedback controller which guarantees that the output y and its derivatives up to order n-1 track a given reference signal y_r and its corresponding derivatives, while keeping all the states bounded.

3 Function Approximation Using Gaussian Radial Basis Functions

The control design presented in this paper employs an RBF neural network to approximate the functions $F(\cdot)$ and $G(\cdot)$ over a compact region of the state space. RBF networks are of the general form $\hat{F}(\cdot) = \theta^T f(\cdot)$, where $\theta \in R^p$ is a vector of adjustable weights and $f(\cdot)$ a vector of Gaussian basis functions. It has been shown that given a smooth function $F : \Omega \mapsto R$, where Ω is a compact subset of R^{m+n} and $\epsilon > 0$, there exists a Gaussian basis function vector $f : R^{m+n} \mapsto R^p$ and a weight vector $\theta^* \in R^p$ such that $|F(x) - \theta^{*T} f(x)| \le$

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$\epsilon \ \forall \ x \in \Omega$. The quantity $F(x) - \theta^{*T} f(x) \stackrel{\text{def}}{=} d(x)$ is called the **network reconstruction error**. The optimal weight vector θ^* defined above is a quantity required only for analytical purposes. Typically θ^* is chosen as the value of θ that minimizes $d(x)$ over Ω. The choice of the Gaussian network parameters used in our control design is motivated by the discussion in [6, Section III]. The update law for the weight vector θ is derived in the next section.

4 Control Design

We start with the following representation for the functions $F(\cdot)$ and $G(\cdot)$, valid for all $x \in Y$ and $z \in Z$, where Y and Z are compact sets defined in Section 4.1.1.

$$\left. \begin{array}{l} F(x,z) = \theta_f^{*T} f(x,z) + d_F(x,z), \\ G(x,z) = \theta_g^{*T} g(x,z) + d_G(x,z) \end{array} \right\} \qquad (2)$$

Assumption 2 *The vectors θ_f^* and θ_g^* belong to known compact subsets $\Omega_f \subset R^{p_1}$ and $\Omega_g \subset R^{p_2}$.*

Typically, some off-line training is done to obtain values θ_{f_0} and θ_{g_0} that result in "good" approximations of the functions F and G over $Y \times Z$. The sets Ω_f and Ω_g are then chosen judiciously as compact sets that contain θ_{f_0} and θ_{g_0}. The fixed optimal weights θ_f^* and θ_g^* in (2) are replaced by their time varying estimates $\hat{\theta}_f$ and $\hat{\theta}_g$, that are adapted during learning. The network approximations associated with these weights are denoted by $\hat{F}$ and $\hat{G}$ respectively.

Assumption 3 $|\hat{G}(\cdot)| \geq k_2 > 0 \ \forall \ x \in Y, \ z \in Z$ and $\hat{\theta}_g \in \hat{\Omega}_g$, *where $\hat{\Omega}_g$ is a compact set that contains Ω_g in its interior.*

4.1 Small Reconstruction Error

Under the assumption of small reconstruction errors, we design an adaptive controller so that the output y tracks the given reference signal y_r. Define $e_i = y^{(i-1)} - y_r^{(i-1)}$, $1 \leq i \leq n$ and $e = [e_1, \ e_2, \ \ldots, \ e_n]^T$. Let

$$\mathcal{Y}(t) = [y(t), \ y^{(1)}(t), \ \ldots, \ y^{(n-1)}(t)]^T$$

$$\mathcal{Y}_r(t) = [y_r(t), \ y_r^{(1)}(t), \ \ldots, \ y_r^{(n-1)}(t)]^T$$

$$\mathcal{Y}_R(t) = [y_r(t), \ y_r^{(1)}(t), \ \ldots, \ y_r^{(n-1)}(t), \ y_r^{(n)}(t)]^T$$

and Y_0 and Y_R be any given compact subsets of R^n and R^{n+1} respectively, such that $\mathcal{Y}(0) \in Y_0$ and $\mathcal{Y}_R(t) \in Y_R$ $\forall \ t \geq 0$. We rewrite (1) as

$$\left. \begin{array}{l} \dot{e} = A_m e + b\{Ke + \theta_f^{*T} f(e + \mathcal{Y}_r, z) \\ \quad + \theta_g^{*T} g(e + \mathcal{Y}_r, z)v + d(e + \mathcal{Y}_r, z, \hat{\theta}_f, \hat{\theta}_g) - y_r^{(n)}\} \\ \dot{z} = A_2 z + b_2 v \end{array} \right\} \qquad (3)$$

where [1] $d(e + \mathcal{Y}_r, z, \hat{\theta}_f, \hat{\theta}_g) = d_F(\cdot) + d_G(\cdot)v$, (A, b) and (A_2, b_2) are controllable canonical pairs that represent chains of n and m integrators, respectively, and K is chosen such that $A_m = A - bK$ is Hurwitz.

Assumption 4 *The system $\dot{\zeta} = H(\zeta, \mathcal{Y}_r)$ has a unique steady-state solution $\bar{\zeta}$. Moreover, with $\tilde{\zeta} = \zeta - \bar{\zeta}$ the system*

$$\begin{array}{rcl} \dot{\tilde{\zeta}} & = & H(\tilde{\zeta} + \bar{\zeta}, e + \mathcal{Y}_r) - H(\bar{\zeta}, \mathcal{Y}_r) \\ & \stackrel{\text{def}}{=} & \tilde{H}(\tilde{\zeta}, e, \mathcal{Y}_r, \bar{\zeta}) \end{array} \qquad (4)$$

has a continuously differentiable function $V_1(t, \tilde{\zeta})$ that satisfies

$$\eta_1 \|\tilde{\zeta}\|^2 \leq V_1(t, \tilde{\zeta}) \leq \eta_2 \|\tilde{\zeta}\|^2$$

$$\frac{\partial V_1}{\partial t} + \frac{\partial V_1}{\partial \tilde{\zeta}} \tilde{H}(\tilde{\zeta}, e, \mathcal{Y}_r, \bar{\zeta}) \leq -\eta_3 \|\tilde{\zeta}\|^2 + \eta_4 \|\tilde{\zeta}\| \|e\|$$

where $\eta_1, \eta_2, \eta_3 > 0$, and $\eta_4 \geq 0$ are independent of $\mathcal{Y}_r$.

Assumption 4 implies that the zero dynamics of (1) are exponentially stable and (1) is minimum phase.

4.1.1 State Feedback:

Let $P = P^T > 0$ be the solution of the Lyapunov equation $PA_m + A_m^T P = -Q$ where $Q = Q^T > 0$, and consider the Lyapunov function candidate

$$V = e^T P e + \tfrac{1}{2} \tilde{\theta}_f^T \Gamma_f^{-1} \tilde{\theta}_f + \tfrac{1}{2} \tilde{\theta}_g^T \Gamma_g^{-1} \tilde{\theta}_g$$

where $\tilde{\theta}_f = \hat{\theta}_f - \theta_f^*$, $\tilde{\theta}_g = \hat{\theta}_g - \theta_g^*$ and $\Gamma_f = \Gamma_f^T > 0$ and $\Gamma_g = \Gamma_g^T > 0$ are gains to be specified later. Taking

$$v = \frac{-Ke + y_r^{(n)} - \hat{F}(\cdot)}{\hat{G}(\cdot)} \stackrel{\text{def}}{=} \psi(e, z, \mathcal{Y}_R, \hat{\theta}_f, \hat{\theta}_g)$$

and using (3), we have

$$\begin{array}{l} \dot{V} = -e^T Q e + 2e^T P b \ d(\cdot) \\ \quad + \tilde{\theta}_f^T \Gamma_f^{-1} [\dot{\hat{\theta}}_f - \Gamma_f \phi_f] + \tilde{\theta}_g^T \Gamma_g^{-1} [\dot{\hat{\theta}}_g - \Gamma_g \phi_g] \end{array} \qquad (5)$$

where $\phi_f = 2e^T P b f(e + \mathcal{Y}_r, z)$ and $\phi_g = 2e^T P b g(e + \mathcal{Y}_r, z)\psi(\cdot)$. Let $\hat{\Omega}_f$ be a compact subset of R^{p_1} that contains Ω_f in its interior. Define

$$\theta = \begin{bmatrix} \theta_f \\ \theta_g \end{bmatrix}, \hat{\theta} = \begin{bmatrix} \hat{\theta}_f \\ \hat{\theta}_g \end{bmatrix}, \tilde{\theta} = \begin{bmatrix} \tilde{\theta}_f \\ \tilde{\theta}_g \end{bmatrix}, \phi = \begin{bmatrix} \phi_f \\ \phi_g \end{bmatrix},$$

$$\Gamma = \text{diag}[\Gamma_f, \Gamma_g], \ \Omega = \Omega_f \times \Omega_g \text{ and } \hat{\Omega} = \hat{\Omega}_f \times \hat{\Omega}_g$$

The parameter adaptation law is chosen as in [4], i.e, $\dot{\hat{\theta}} = \text{Proj}(\hat{\theta}, \phi)$, where $\text{Proj}(\hat{\theta}, \phi) = \Gamma\phi$ for $\hat{\theta} \in \Omega$ and is modified outside Ω to ensure that

$$\tilde{\theta}^T \Gamma^{-1} [\dot{\hat{\theta}} - \Gamma\phi] \leq 0 \qquad (6)$$

[1] The dependence on $\hat{\theta}_f$ and $\hat{\theta}_g$ comes through v.

and $\hat{\theta}(t)$ belongs to a compact set Ω_δ $\forall\ t \geq 0$, where $\hat{\Omega} \supset \Omega_\delta \supset \Omega$.

We assume that $e(0)$ and $z(0)$ belong to known compact subsets $E_0 \subset R^n$ and $Z_0 \subset R^m$ and let $c_1 = \max_{e\in E_0} e^T Pe$. Choose $c_4 > c_1$ and define $E \stackrel{\text{def}}{=} \{e^T Pe \leq c_4\}$ and $Y \stackrel{\text{def}}{=} \{e + \mathcal{Y}_r | e \in E, \mathcal{Y}_r \in Y_R\}$. Let Z be a compact subset of R^m such that Z_0 is in the interior of Z and

$$z(0) \in Z_0 \text{ and } e(t) \in E\ \forall\ t \geq\ 0\ \Rightarrow\ z(t) \in Z\ \forall\ t \geq 0.$$

The RBF networks are used to approximate $F(\cdot)$ and $G(\cdot)$ over the compact set $Y \times Z$.

Define Ω_{δ_f} and Ω_{δ_g} by $\Omega_\delta = \Omega_{\delta_f} \times \Omega_{\delta_g}$ and let

$$c_2 = \max_{\theta_f^* \in \Omega_f, \hat{\theta}_f \in \Omega_{\delta_f}} \tfrac{1}{2}(\hat{\theta}_f - \theta_f^*)^T \Gamma_f^{-1} (\hat{\theta}_f - \theta_f^*),$$

$$c_3 = \max_{\theta_g^* \in \Omega_g, \hat{\theta}_g \in \Omega_{\delta_g}} \tfrac{1}{2}(\hat{\theta}_g - \theta_g^*)^T \Gamma_g^{-1} (\hat{\theta}_g - \theta_g^*).$$

The adaptation gains Γ_f and Γ_g are chosen large enough to ensure that $c_4 - c_1 > c_2 + c_3$. This is different from [1] where the adaptation gain is not required to be large. This is because, in the present case, the compact sets Ω_f and Ω_g to which the optimal weights θ_f^* and θ_g^* belong depend on the set E, because the approximation of F and G is done over the set $Y \times Z$. Hence the set Y has to be defined prior to, and consequently, independent of the sets Ω_f and Ω_g. This requires making the adaptation gains large.

Let $d = max\|d(e + \mathcal{Y}_r, z, \hat{\theta}_f, \hat{\theta}_g)\|$, where the maximization is done over all $e + \mathcal{Y}_r \in Y, z \in Z, \hat{\theta}_f \in \Omega_{\delta_f}$ and $\hat{\theta}_g \in \Omega_{\delta_g}$. Using (5) and (6), $\forall e \in E$, we have

$$\dot{V} \leq -e^T Qe + k_d d, \text{where } k_d = \max_{e\in E} 2\|e\|\ \|P\ b\| \quad (7)$$

If $d < d^* = k(c_4 - c_3 - c_2)/k_d$, where $k = \frac{\lambda_{min}(Q)}{\lambda_{max}(P)}$, then $\dot{V} < 0$ on $\{V = c_4\} \cap\ \Omega_\delta$. Thus the set $\{V \leq c_4\} \cap\ \Omega_\delta$ is positively invariant for all $d < d^*$. Inside this set, $e \in E$. As long as $e \in E$, z will remain in Z. Thus the trajectory $(e, z, \hat{\theta})$ is trapped inside the set $R_s = \{e \in E\} \times \{z \in Z\} \times \{\hat{\theta} \in \Omega_\delta\}$. Hence all the states are bounded and from (7), the mean-square tracking error is of the order $O(d)$.

4.1.2 Output Feedback: To implement the controller developed in the previous section using output feedback, we replace the states e by their estimates $\hat{e}$ provided by a high gain observer (HGO). The control is saturated outside a compact region of interest to prevent the peaking induced by the HGO [4]. The HGO used to estimate the states is the same one used in [4] and is described by the following equations,

$$\left.\begin{aligned} \dot{\hat{e}}_i &= \hat{e}_{i+1} + \tfrac{\alpha_i}{\epsilon^i}(e_1 - \hat{e}_1), \quad 1 \leq i \leq n-1 \\ \dot{\hat{e}}_n &= \tfrac{\alpha_n}{\epsilon^n}(e_1 - \hat{e}_1) \end{aligned}\right\} \quad (8)$$

where $\epsilon > 0$ is a design parameter that will be specified shortly. The positive constants α_i are chosen such that the roots of $s^n + \alpha_1 s^{n-1} + \cdots + \alpha_{n-1}s + \alpha_n = 0$ have negative real parts. Let $\xi_i = \frac{e_i - \hat{e}_i}{\epsilon^{n-i}}$, $1 \leq i \leq n$, $\xi = [\xi_1, \ldots, \xi_n]^T$ and $V_\xi = \xi^T \bar{P} \xi$, where $\bar{P} = \bar{P}^T > 0$ is the solution of the Lyapunov equation $\bar{P}(A - HC) + (A - HC)^T \bar{P} = -I$. Boundedness of all signals of the closed-loop system can be proved by an argument similar to the one in Section 4.1.1. First, it is not difficult to show that for all $(e, \hat{\theta}) \in \{V \leq c_4\} \cap\ \Omega_\delta$, there exist constants $c_6,\ c_7 > 0$ such that the sets $\{V_1 \leq c_6\}$ and $\{V_\xi \leq c_7\epsilon^2\}$ are positively invariant. Next, using the results of [1, Section 5], for all $(e, \hat{\theta}, \tilde{\zeta}, \xi)$ belonging to the set

$$R = \{\{V \leq c_4\} \cap \Omega_\delta\} \times \{V_1 \leq c_6\} \times \{V_\xi \leq c_7\epsilon^2\},$$

the derivative of V satisfies

$$\dot{V} \leq -e^T Qe + k_\epsilon \epsilon + k_d d, \text{ where } k_\epsilon > 0.$$

Hence for all $d < d^* = \frac{k(c_4 - c_3 - c_2)}{2k_d}$ and $\epsilon < \epsilon^* = \frac{k(c_4 - c_3 - c_2)}{2k_\epsilon}$, $\dot{V} < 0$ on $\{V = c_4\} \cap\ \Omega_\delta$, and the set R is positively invariant. Using the difference in speeds between the slow and fast variables and the fact that $\dot{V}_\xi \leq -(1/2\epsilon)\|\xi\|^2$ outside $\{V_\xi \leq c_7\epsilon^2\}$ it can be shown that the trajectory enters the set R during the time interval $[0, T(\epsilon)]$, where $T(\epsilon) \to 0$ as $\epsilon \to 0$. Hence, as in the previous case, for sufficiently small d and ϵ, all the states are bounded and the mean-square tracking error is of the order $O(\epsilon + d)$.

4.2 Reconstruction Error With a Known Bound

We design an additional robustifying control component to make the mean-square tracking error arbitrarily small, irrespective of the bound on the disturbance d, provided this bound is known. Let

$$v = \frac{-Ke + y_r^{(n)} - \hat{F}(e + \mathcal{Y}_r, z, \hat{\theta}_f) + v_1}{\hat{G}(e + \mathcal{Y}_r, z, \hat{\theta}_g)} \quad (9)$$

Assume $\|d(\cdot)\| \leq \rho(e, z) + k_v\|v_1\|$, where $0 \leq k_v < 1$ and ρ and k_v are known. Take $\eta(e, z) \geq \rho(e, z)$ and define $s = 2e^T Pb$,

$$\psi_r(e, z) = \begin{cases} -\frac{\eta(e,z)}{(1-k_v)} \cdot \frac{s}{|s|} & \text{for} \quad \eta(e, z)|s| \geq \mu \\ -\frac{\eta^2(e,z)}{(1-k_v)} \cdot \frac{s}{\mu} & \text{for} \quad \eta(e, z)|s| < \mu \end{cases} \quad (10)$$

The control is taken as

$$\psi(\cdot) = \frac{-Ke + y_r^{(n)} - \hat{F}(e + \mathcal{Y}_r, z, \hat{\theta}_f) + \psi_r(e, z)}{\hat{G}(e + \mathcal{Y}_r, z, \hat{\theta}_g)} \quad (11)$$

As before, e is replaced by its estimate $\hat{e}$ and the control is saturated outside a compact region of interest. The arguments of the preceding section can be extended to show that $\exists\ \epsilon^* > 0$ and $\mu^* > 0$ such that $\forall\ 0 < \epsilon < \epsilon^*$ and $0 < \mu < \mu^*$, all signals are bounded and the mean-square tracking error is of the order $O(\epsilon + \mu)$, where the design parameters ϵ and μ can be made arbitrarily small.

5 Simulations

In this section, two simulations are presented to illustrate the points made in the earlier sections. In the first simulation, we show the effect of changing various design parameters on the tracking error. In the second, we attempt to justify the need to adapt for the network's weights. The plant used in all these simulations is the same one used in [6, 7], namely $\ddot{y} = F(y, \dot{y}) + G(y)u$, where

$$F(y, \dot{y}) = 16\frac{sin(4\pi y)}{4\pi y}\left(\frac{sin(\pi\dot{y})}{\pi\dot{y}}\right)^2 \text{ and}$$
$$G(y) = 2 + sin(3\pi(y - 0.5))$$

5.1 Simulation 1

The plant output is required to track a reference signal y_r that is the output of a low -pass filter with transfer function $(1 + s/10)^{-3}$, driven by a unity amplitude square wave input with frequency 0.4 Hz and a time average of 0.5. Since $m = 0$, there is no need to augment integrators at the system's input. Let $\tilde{Y} \stackrel{\text{def}}{=} [-1, 1] \times [-3, 3]$. We use 2 RBF networks to approximate the functions $F(y, \dot{y})$ and $G(y)$ over $\tilde{Y}$. The networks have 48 Gaussian nodes with variance[2] $\sigma^2 = 4\pi$ spread over a regular grid that covers $\tilde{Y}$. Off-line training is done to obtain weights θ_{f_0} and θ_{g_0} that result in "optimal" approximations of the functions F and G. However, the reconstruction errors are still quite large in this case, at some points being comparable to the value of the function itself. Based on the values of θ_{f_0} and θ_{g_0}, the sets Ω_f and Ω_g in Assumption 1 are taken as $[\theta_{f_0} - 0.1, \theta_{f_0} + 0.1]$ and $[\theta_{g_0} - 0.1, \theta_{g_0} + 0.1]$, where the addition and subtraction are done component wise. The adaptation gains Γ_f and Γ_g are taken for simplicity as $10^3 I$. The values of the other design parameters are $\eta = 40$ and $k_v = 0.7$. The initial condition $x(0)$ is taken as (-0.5,2.0). Fig(1a) shows the tracking

[2]See [6] for a definition of this term in relation to RBF networks

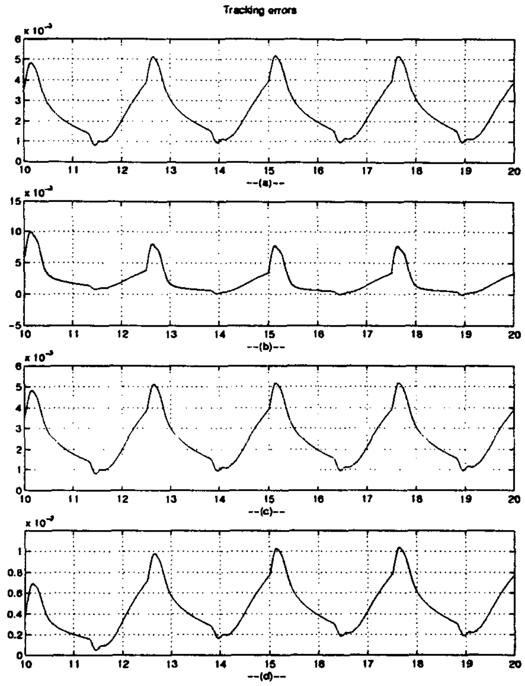

Figure 1: (a)State feedback $\mu = 0.5$ (b) Output feedback, $\epsilon = 10^{-3}$, $\mu = 0.5$ (c) Output feedback $\epsilon = 10^{-4}$, $\mu = 0.5$ (d) Output feedback $\epsilon = 10^{-4}$, $\mu = 0.1$

error for the state feedback case with $\mu = 0.5$, Fig(1b) is the output feedback case with $\epsilon = 10^{-3}$, Fig(1c) with ϵ reduced to 10^{-4}, and Fig(1d) with μ reduced to 0.1. The simulation illustrates several points : (a) by using a robustifying component, it is possible to obtain reasonable performance even with networks that give large reconstruction errors; (b) as ϵ is decreased, we recover the performance obtained under state feedback; and (c) an n-fold decrease in μ results in approximately an n-fold decrease in the tracking error. Thus, by decreasing μ, we can meet more stringent requirements on the tracking error.

5.2 Simulation 2

The initial weights obtained by off-line training may not be close to their optimal values. This might, for example, be the case when the off-line training is done (based) on a nominal model that differs considerably from the actual one. For definiteness, suppose the function $F(y, \dot{y})$ is any one of the functions

$$F(y, \dot{y}) = k_1\frac{sin(4\pi(y - k_2))}{4\pi(y - k_2)}\left(\frac{sin(\pi(\dot{y} - k_3))}{\pi(\dot{y} - k_3)}\right)^2$$

where $k_1 \in [15, 17]$, $k_2 \in [-0.5, 0.5]$ and $k_3 \in [-1, 1]$ and that a nominal model is the one used before. For simplicity, we take $G(y) = 1$. Further, the reference signal is taken as $y_r = 0.4$. This time, we use

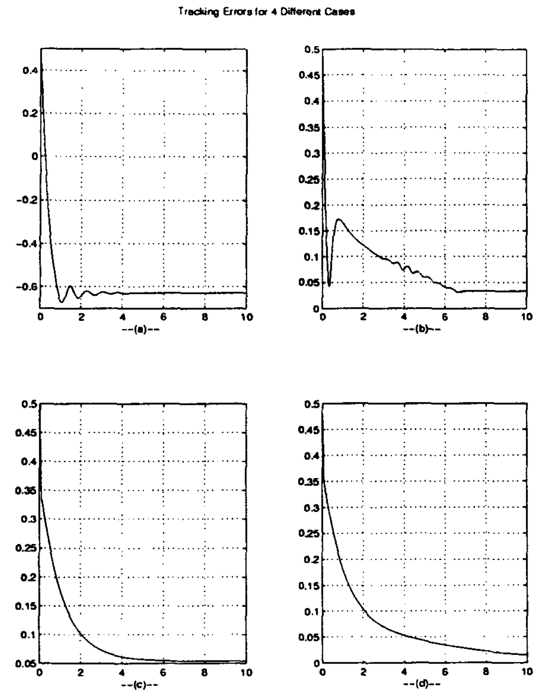

Figure 2: (a) No adaptation for weights, no robust control (b) Only adaptation for weights (c) Only robust control (d) Adaptation for weights and robust control.

an RBF network with 192 Gaussian nodes to "construct" the function $F(\cdot)$, with the parameters of the network chosen as before. Based on the nominal model, we do off-line training to obtain initial estimates θ_{f_0} and θ_{g_0}. For the purpose of simulation, the values of k_1, k_2 and k_3 are taken to be 17, 0.4 and 0 respectively. This choice ensures that with the "nominal" weights θ_{f_0}, the reconstruction error is quite large in the region of the state space where the reference lies. The values of the parameters used in the design are $\Gamma_f = 10^3 I, \epsilon = 10^{-4}$, $\eta = 20$, $k_v = 0$ and $\mu = 0.2$. The initial condition $x(0)$ is taken as $(0.9, -2.75)$. Fig(2a) shows the tracking error for the case when there is no adaptation for the weights, that is, $\Gamma_f = 0$ and no robust control component, Fig(2b) for the case when the weights are adapted but there is no robust component, Fig(2c) for the case when the weights are not adapted but there is a robust component, and Fig(2d) for the case when the weights are adapted and a robust component is used. In the first case the tracking error is quite large because we simply do a crude cancelation of the network nonlinearity based on a nominal model. When we start adapting for the weights, the difference between the function F and its estimate $\hat{F}$ provided by the network decreases and hence the tracking error also decreases. However, even with the network providing its "best" approximation, there is a residual error. In the case where we simply use robust control, the performance shows an improvement over the first case and is almost comparable to the error in the second case. Finally, in the case where we do both adaptation and robust control, the network reconstruction error decreases and the robust component handles this smaller error better. Thus the tracking error is the smallest in this case.

6 Conclusions

An adaptive output feedback scheme that uses RBF neural networks has been presented. The method is based on the results of [1] and uses RBF networks to approximately construct the system nonlinearities. The reconstruction errors of the networks are not required to be small, thus allowing for the use of lower-order networks. Another merit of the scheme is the use of the HGO to estimate the output derivatives, thus dispensing with the requirement of availability of all the states.

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