Proceedings of the American Control Conference
San Diego, California • June 1999

Longitudinal Adaptive Control of a Platoon of Vehicles

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Abstract

A technique for the longitudinal control of a platoon of automated vehicles is presented. A nonlinear model is used to represent the vehicle dynamics of each vehicle within the platoon. The controlled vehicle is assumed to be capable of measuring (or estimating) necessary dynamical information from the vehicle immediately in front of it by its on-board sensors. The computer in the vehicle processes the measured data and generates proper throttling and braking actions to follow the vehicle in front at a safe distance. Simulations are presented for the case of a platoon of four cars following a leader.

1 Introduction

The subject of design and analysis of various longitudinal control laws for automated highway systems (AHS) has been studied extensively since the late 1960's. The goal is to significantly increase the traffic capacity of existing highways through vehicle and roadway automation. Furthermore, since many of today's automobile accidents are caused by human error, automating the driving process may actually increase highway safety. In such a system, vehicles will be driven automatically with on-board lateral and longitudinal controllers. The lateral controller will be used to steer the vehicle around corners, make lane changes, and perform additional steering tasks. The longitudinal controller will be used to maintain a steady velocity if the vehicle is traveling alone (conventional cruise control) or follow a lead vehicle at a safe distance (vehicle following). In this paper, we discuss the application of the adaptive control technique of [2] to the vehicle following problem. A simplified nonlinear longitudinal powertrain model is used for designing the controller. The vehicle parameters are partially known or completely unknown and are adapted for. We assume that the following measurements are available to the vehicle's sensors (i) the relative distance [1] between the controlled car and the car in front of it and (ii) the forward velocity of the controlled car. The other quantities of interest, namely the relative velocity, relative acceleration and the acceleration/deceleration of the controlled car, are estimated from the measured quantities. The idea of replacing measured quantities by their estimates has also been used in earlier works. For example [9] mentions the possibility of "direct computation" of relative velocity and acceleration using the measured value for the relative distance. However, in [9] (i) the control objective is different from the one we consider here, (ii) the model used is a simplified one where all parameters are assumed exactly known and there are no disturbances and (iii) no analysis is presented for the case where estimates are used in feedback. Similarly, [1] uses an estimate of the leading vehicle's acceleration in the control. However, the measured quantities still include (in addition to the relative distance and the controlled vehicle's velocity) the relative velocity between the controlled and leading vehicles, and the acceleration/deceleration and propulsion force of the controlled vehicle.

[1] Referred to as the intervehicle spacing in the next section.

2 Longitudinal Vehicle Model

A widely proposed strategy for effectively increasing traffic throughput on existing highways through automation is to group the controlled vehicles into tightly spaced vehicle group formations called **platoons** [12]. A configuration of a platoon of N+1 vehicles is shown in Fig 1. The lead vehicle is numbered 0 and the ith follower (henceforth referred to as the ith vehicle) is numbered i. L_i denotes the length of the ith vehicle and x_i its position. Let $\delta_i = x_{i-1} - x_i - L_i$ for $i = 1,2,\ldots$, N. δ_i is the intervehicle spacing between the $(i-1)$th and ith vehicles. In developing a model for the system, we assume that the road surface is horizontal and that all vehicles travel in the same direction at all times. ¿From Newton's Second Law, the relationship between the acceleration of the ith vehicle, its propulsion force, and the drag forces acting on it can be derived as

$$m_i \ddot{x}_i = f_i - k_{d_i} \dot{x}_i^2 - d + d_{1_i}(t) \qquad (1)$$

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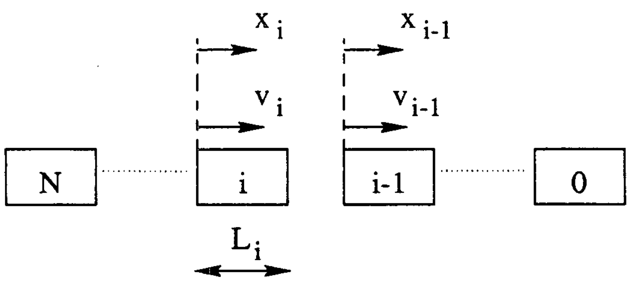

Figure 1: A platoon of N+1 vehicles

where m_i is the mass of the vehicle, $\ddot{x}_i$ its acceleration, f_i the propulsion force, $k_{d_i}\dot{x}_i^2$ the aerodynamic drag force, d a nominal constant mechanical drag and $d_{1_i}(t)$ the resultant of the external disturbances (such as wind gust,...etc.) The propulsion system which represents the engine dynamics of the vehicle can be modeled as a first order system [1]

$$\dot{f}_i = \frac{1}{\tau_i}(-f_i + u_i) + d_{2_i}(t) \qquad (2)$$

where τ_i denotes the vehicle's engine time-constant, u_i is the throttle/brake input and $d_{2_i}(t)$ is a disturbance term (possibly due to engine transmission variations, ... etc.) This model differs from the one in [1] in that both the engine time-constant and the mechanical drag term are independent of the vehicle's velocity. However, we note that the effects of neglecting this dependence can be incorporated into the disturbance terms $d_{1_i}(t)$ and $d_{2_i}(t)$. The constants k_{d_i}, m_i and τ_i are unknown but belong to known compact subsets of R^1.

3 Control Objective and Design

The dynamics of the ith vehicle maybe described by the state vector $[\delta_i, v_i, f_i]^T$, where $v_i = \dot{x}_i$ is the ith vehicle's velocity. With this choice of state variables, (1) and (2) maybe rewritten as

$$\left.\begin{array}{lcl} \dot{\delta}_i & = & v_{i-1} - v_i, \\ \dot{v}_i & = & (f_i - k_{d_i}v_i^2 - d + d_{1_i}(t))/m_i \\ \dot{f}_i & = & (-f_i + u_i)/\tau_i + d_{2_i}(t) \end{array}\right\} \qquad (3)$$

for $1 \leq i \leq N$. The control objective is to design u_i in such a way that the intervehicle spacing δ_i tracks a desired reference. It is well known (see for example [8, 10]) that for the case where the desired intervehicle spacing is constant, asymptotic platoon stability can be guaranteed only if the lead vehicle is transmitting its velocity and acceleration to all other vehicles in the platoon. This approach yields stable platoons with small intervehicle spacings at the cost of introducing and maintaining continuous intervehicle communication with high reliability and small delays. In [3], it is shown that platoon stability can be recovered in a non-cooperative or autonomous operation if a speed dependent spacing policy is adopted, which incorporates a constant time headway in addition to the constant distance. This takes the form $\delta_{d_i} = \lambda v_i + \lambda_0$, where δ_{d_i} is the desired intervehicle spacing and λ and λ_0 are suitably chosen positive constants. The parameter λ is the **time headway** and its effect is to introduce more spacing between the ith and $(i-1)$th vehicles as the velocity of the ith vehicle increases, which intuitively makes sense. Following [11], we set λ_0 to zero, which basically allows for the minimum desired distance between two adjacent vehicles to be zero provided the vehicle that is following has zero velocity. With this choice, we define the plant output as $y_{p_i} = \delta_i - \lambda v_i$. The control objective is thus to regulate y_{p_i} to zero. Differentiating the output twice and making use of (3), the following error equation is obtained

$$\ddot{y}_{p_i} = \ddot{\delta}_i + \theta_i^T[F_i(v_i, \dot{v}_i) + Gu_i] + D_i(t) \qquad (4)$$

where $\theta_i = [k_{d_i}/m_i, 1/\tau_i, k_{d_i}/(m_i\tau_i), 1/(m_i\tau_i)]^T$, $F_i(\cdot) = [2\lambda v_i\dot{v}_i, \lambda\dot{v}_i, \lambda v_i^2, \lambda d]^T$, $G = [0,0,0,-\lambda]^T$ and $D_i(t) = -\lambda(\dot{d}_{1_i}/\tau_i + \dot{d}_{1_i} + d_{2_i})/m_i$. From the knowledge of the intervals in which k_{d_i}, m_i and τ_i lie, it is possible to calculate the compact subset of R^4 to which θ_i belongs. By defining $Y_{p_i} = [y_{p_i}, \dot{y}_{p_i}]^T$, it is possible to rewrite (4) as

$$\dot{Y}_{p_i} = A_m Y_{p_i} + b\{KY_{p_i} + \ddot{\delta}_i + \theta_i^T[F_i(v_i, \dot{v}_i) + Gu_i] + D_i(t)\},$$

where (A, b) is a controllable canonical pair that represents a chain of two integrators and K is chosen such that $A_m = A - bK$ is Hurwitz. To estimate $\dot{\delta}_i$, $\ddot{\delta}_i$ and $\dot{v}_i$, we use two high-gain observers, driven by δ_i and v_i respectively. Denote the estimates by $\hat{\dot{\delta}}_i$, $\hat{\ddot{\delta}}_i$ and $\hat{\dot{v}}_i$ respectively. The high-gain observers are described by the following equations

$$\begin{aligned} \dot{w}_{1_i} &= w_{2_i} + \beta_1(\delta_i - w_{1_i})/\epsilon \\ \dot{w}_{2_i} &= w_{3_i} + \beta_2(\delta_i - w_{1_i})/\epsilon^2 \\ \dot{w}_{3_i} &= \beta_3(\delta_i - w_{1_i})/\epsilon^3 \\ \hat{\dot{\delta}}_i &= w_{2_i}, \ \hat{\ddot{\delta}}_i = w_{3_i} \end{aligned}$$

and

$$\begin{aligned} \dot{z}_{1_i} &= z_{2_i} + \alpha_1(v_i - z_{1_i})/\epsilon \\ \dot{z}_{2_i} &= \alpha_2(v_i - z_{1_i})/\epsilon^2 \\ \hat{\dot{v}}_i &= z_{2_i} \end{aligned}$$

where $\epsilon > 0$ and the α's and β's are chosen such that the roots of $s^2 + \alpha_1 s + \alpha_2 = 0$ and $s^3 + \beta_1 s^2 + \beta_2 s + \beta_3 = 0$ have negative real parts. Let $\hat{\dot{y}}_{p_i} = \hat{\dot{\delta}}_i - \lambda\hat{\dot{v}}_i$, $\hat{Y}_{p_i} = [y_{p_i}, \hat{\dot{y}}_{p_i}]^T$ and assume that an upper bound on the term $D_i(t)$ is known. Then the control u_i is designed as

$$u_i = \frac{-\hat{\ddot{\delta}}_i - \hat{\theta}_i^T F_i(v_i, \hat{\dot{v}}_i) - K\hat{Y}_{p_i} + v_{r_i}}{\hat{\theta}_i^T G} \qquad (5)$$

where $\hat{\theta}_i$ is an estimate of θ_i and v_{r_i} is a robustifying component designed using the Lyapunov redesign technique,e.g., [6, Section 13.1]. The control u_i is saturated outside a compact set of interest to prevent the peaking induced by the high-gain observers [4].[2] The parameter adaptation law is chosen as in [2]. In particular, let $P = P^T > 0$ be the solution of the Lyapunov equation $PA_m + A_m^T P = -I$. Define $\phi_i = 2Y_{p_i}^T Pb[F_i(v_i, \dot{v}_i)+Gu_i]$ and let $\Gamma = \Gamma^T > 0$. Then the adaptation law is chosen as $\dot{\hat{\theta}}_i = \text{Proj}(\hat{\theta}, \phi)$, where $\text{Proj}(\hat{\theta}_i, \phi_i)=\Gamma\phi_i$ for $\hat{\theta}_i \in \Omega$ and is modified outside Ω to ensure that $\tilde{\theta}_i^T \Gamma^{-1}[\dot{\hat{\theta}}_i - \Gamma\phi_i] \leq 0$ and $\hat{\theta}_i(t)$ belongs to a compact set Ω_δ $\forall$ $t \geq 0$, where $\Omega_\delta \supset \Omega$. Starting with the Lyapunov function candidate

$$V_i = Y_{p_i}^T P Y_{p_i} + \tfrac{1}{2}\tilde{\theta}_i^T \Gamma^{-1} \tilde{\theta}_i,$$

and proceeding along the lines of [2], it is possible to show ultimate boundedness of the spacing deviation error. Though the proof in [2] was done for the single-output case, an extension to the multi-output case is not very difficult, and has been addressed, for example, in [7]. It is worth mentioning that the proof in [2] only guarantees the boundedness of y_{p_i} and $\dot{y}_{p_i}$. To argue boundedness of δ_i, $\dot{\delta}_i$, $\ddot{\delta}_i$, v_i and $\dot{v}_i$, we first assume that there exist achievable bounds on the leading vehicle's velocity v_0 [11] and acceleration $\dot{v}_0$ [3]. Noting that

$$\lambda\dot{v}_1 + v_1 = v_0 - \dot{y}_{p_1}$$

and that $\dot{y}_{p_1}$ is bounded and $\lambda > 0$, we see that v_1 is bounded. Extending this argument inductively shows that v_i is bounded for all i. Since $y_{p_i} = \delta_i - \lambda v_i$, boundedness of δ_i follows. Furthermore, since each v_i is bounded, so is $\dot{\delta}_i = v_{i-1} - v_i$. From $\dot{y}_{p_i} = \dot{\delta}_i - \lambda\dot{v}_i$, each $\dot{v}_i$ is bounded. And finally, since $\ddot{\delta}_i = \dot{v}_{i-1} - \dot{v}_i$, each $\ddot{\delta}_i$ is bounded.

4 Simulations

In this section, we present two sets of simulations for a platoon of five cars. In all simulations, we assume that all vehicles are initially traveling at a velocity of 15 m/s. The lead vehicle's velocity, acceleration and jerk profiles are shown in Fig 2. We assume that $m_i \in [1100, 1550]kg$, $\tau_i \in [0.15, 0.25]s$, $k_{d_i} \in [0.1, 0.5]Ns^2/m^2$ and $d = 100N$. These values are chosen to be the same as or close to the ones in [1, 11]. For the first set of simulations, we use a value of $\lambda = 0.9$ and for the second $\lambda = 0.2$. The particular values for λ are explained in some detail below.

[2] For the purpose of simulations, the control is saturated at a value slightly higher than the observed value under state feedback.

4.1 Simulation 1

The value of $\lambda = 0.9$ is based on the *California rule of thumb*, [3, 11], which suggests an intervehicle spacing of one vehicle length for every 10 m.p.h. Assuming an average vehicle length of 4 m, this translates to a value of $\lambda = 0.9$ In all simulations, we assume the following values for the vehicle parameters, $m_1 = 1300$, $\tau_1 = 0.16$, $k_{d_1} = 0.3$, $m_2 = 1400$, $\tau_2 = 0.22$, $k_{d_2} = 0.35$, $m_3 = 1200$, $\tau_3 = 0.18$, $k_{d_3} = 0.2$, $m_4 = 1350$, $\tau_4 = 0.24$ and $k_{d_4} = 0.45$. We assume that d_{2_i} is identically zero, but d_{1_i}/m_i is as shown in Fig 3. The disturbance profiles are similar to, though not identical to the ones in [1]. In particular, they are "smooth" functions of time. Fig 4 shows the velocity and acceleration profiles for the following vehicles, the spacing deviation errors, and their positions relative to the leader for the case when no robustifying control is used. Fig 5 is for the case where a robustifying control is used. The spacing deviation error shows a marked decrease in this case. The spacing deviations do not exceed 1.6 cm in magnitude. The above results compare favorably with the results of [11, 1]. It is worth mentioning however, that the spacing policy in [1] is different from the one we adopt here. The spacing deviation errors reported above are also of the same order of magnitude as in [8, 5], where the spacing deviation errors are between 1 and 10 cm. However, as with [1], the results are not directly comparable owing to differences in the vehicle model and/or the spacing policy adopted.

4.2 Simulation 2

The California rule of thumb takes into account human reaction times and delays [3]. In automatic vehicle following, human delays are eliminated and we can afford to have a smaller time headway without affecting safety. In [3], based on a worst case stopping scenario, where the lead vehicle is assumed to be at full deceleration and the following vehicle is at full acceleration at the instant the stop maneuver commences, a value of λ in the range of 0.1 to 0.2 is obtained. For this simulation, we assume $\lambda = 0.2$, $d_{1_i}(t)$ is not zero and a robustifying component is used. Fig 6 shows the results for this case.

5 Conclusions

We have presented a new technique for the longitudinal control problem. Good performance has been achieved in the presence of parameter uncertainties and unknown time-varying disturbances. The main contribution of this method is the use of high-gain observers to reduce the number of sensor measurements. In particular, we do not require direct measurement of the relative velocity or acceleration between the controlled and leading vehicles or the controlled vehicle's acceler-

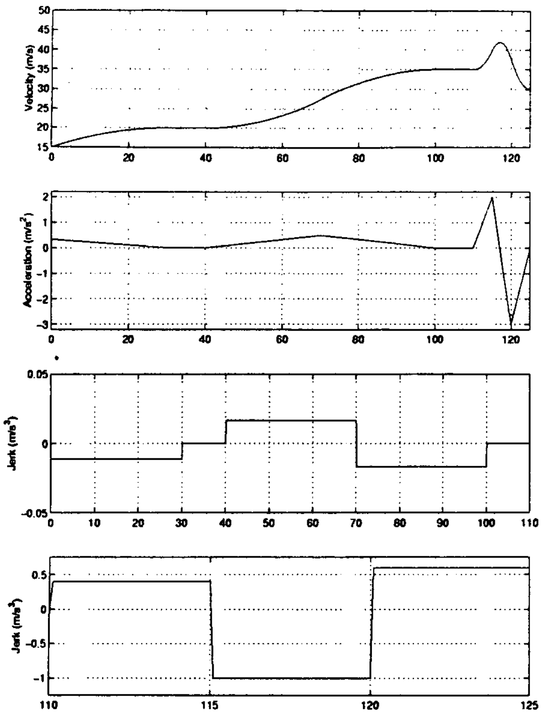

Figure 2: Velocity, acceleration and jerk profiles for the leader.

ation. The spacing deviation errors reported are of the same order of magnitude as in [1, 5, 8, 11].

Acknowledgment: This work was supported in part by NSF under grants EEC9700741 and ECS9703742.

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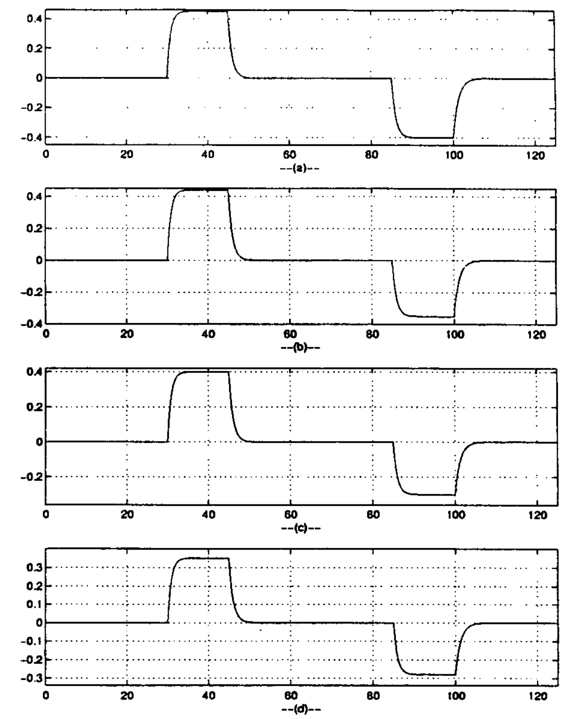

Figure 3: (a) $d_{1_1}(t)$, (b) $d_{1_2}(t)$, (c) $d_{1_3}(t)$, (d) $d_{1_4}(t)$; $d_{2_i}(t) = 0$.

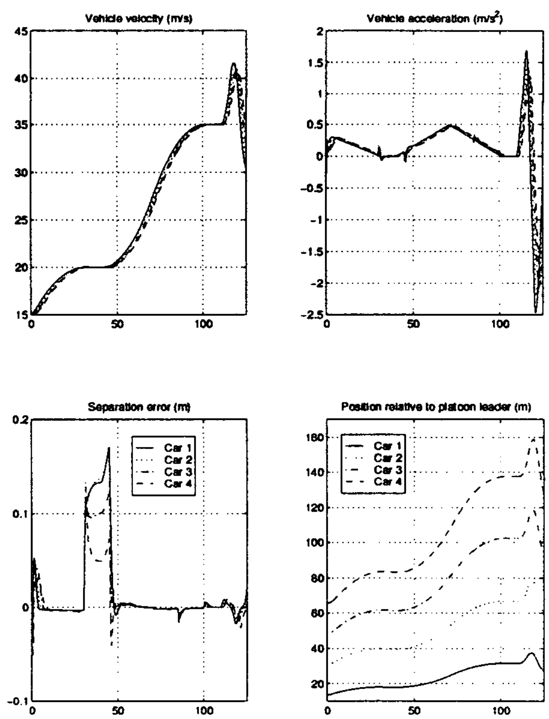

Figure 4: $D_i(t) \neq 0$, robustifying component not included.

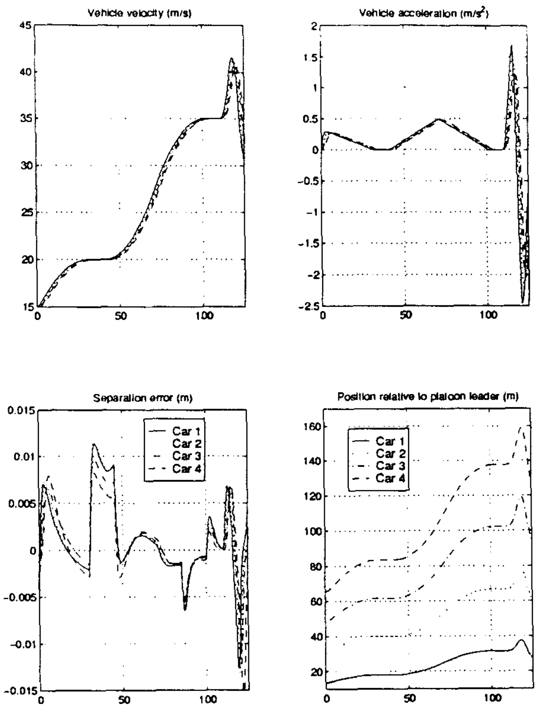

Figure 5: $D_i(t) \neq 0$, robustifying component included, $\lambda = 0.9$.

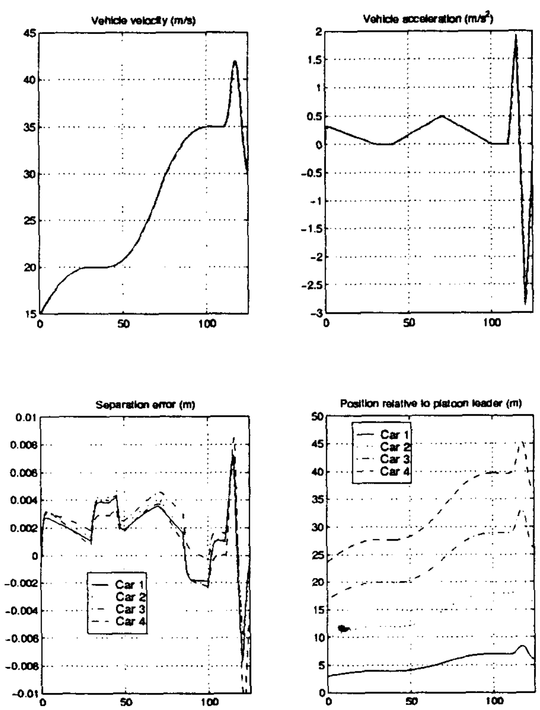

Figure 6: $D_i(t) \neq 0$, robustifying component included, $\lambda = 0.2$.

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